Dynamic Pricing and Inventory Management under Fluctuating Procurement Costs

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Motivation

**HP’s Challenge:** DRAM memory procurement cost dropped by 90% in 2001 and tripled in 2002 (Nagali et al. 2008).
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HP’s Solution:

- Procurement Risk Management (PRM) Program
  - Combined sourcing channels: spot, short- and long- term contracts.
  - $425 million cost reduction over a 6-year period.
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  - $425 million cost reduction over a 6-year period.

- Portfolio Management Process
  - Regular price reviews and adjustments.
  - Price changes in response to production and supply chain costs, as well as global economic conditions, including currency volatility.
Motivation (Cont’d)

- Combined multi-sourcing and dynamic pricing strategy is ubiquitous under procurement cost fluctuation.
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- Executed by separated units of a firm (procurement and marketing).
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- **Goal of our paper:** To understand how dynamic pricing and dual-sourcing strategies can be coordinated under demand uncertainty and procurement cost fluctuation.
Research Questions

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3. What is the relationship between dynamic pricing and dual-sourcing?
Outline

- Related Literature
- Model
- Impact of Cost Volatility
- Strategic Relationship between Dynamic Pricing and Dual-Sourcing
- Conclusion: Takeaway Insights
Literature Review
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- Inventory management under fluctuating costs:
  - Kalymon (1971),
  - Berling and Martínez-de-Albéniz (2011),
  - Chen et al. (2013).

Joint price and inventory control:
- Federgruen and Heching (1999),
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Our paper: Joint pricing and inventory management under demand uncertainty, cost fluctuation, and dual-sourcing.
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Model Formulation: Basics

- $T$—period stochastic inventory system, labeled backwards, with discount factor $\alpha \in (0, 1)$.

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  - Spot market: immediate delivery.
  - Forward-buying contract: postponed delivery.
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- No inventory resale:
  - No room for arbitrage.
Spot-Market Price Fluctuation

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- \( \xi_t \): The random perturbation in the cost dynamics.

- \( s_t(\cdot, \cdot) > 0 \text{ a.s., and } s_t(\hat{c}_t, \xi_t) \succeq_{s.d.} s_t(c_t, \xi_t) \text{ for any } \hat{c}_t > c_t. \)
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- Examples: GBMs, mean-reverting processes.
Forward-Buying Contract

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- Forward-buying contract: \((f_t, q_t)\):
  - The firm pays \(f_t q_t\) to the supplier in period \(t^e\);
  - The supplier delivers \(q_t\) to the firm in period \(t^e\);
  - For technical tractability, \(t^e = t - 1\).
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- \(f_t = \gamma c_t / \alpha\).
  - Effective unit cost: \(\gamma c_t\).
  - In reality, \(f_t = F_t(c_t)\) is determined through bilateral negotiations.
  - Most results hold for \(f_t = F_t(c_t)\), where \(F_t(\cdot)\) is a positive increasing function of \(c_t\).
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- The contract cannot be traded in the derivatives market.
  - Focus on the operational effect of forward-buying.
Demand Model

\[ D_t(p_t) = d(p_t) + \epsilon_t. \]

- \( \epsilon_t \): independent continuous random variables, with \( \mathbb{E}\{\epsilon_t\} = 0 \).

- \( d(\cdot) \): strictly decreasing function of \( p_t \), with a strictly decreasing inverse \( p(\cdot) \) in the expected demand \( d_t \) and \( D_t(p_t) \geq 0 \) a.s.
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**Assumption 1**

\[ R(d_t) := p(d_t)d_t \] is continuously differentiable and strictly concave.
Sequence of Events

- The firm reviews inventory $I_t$ and spot market price $c_t$. 

Demand $D_t$ realized, revenue collected.

Net inventory fully carried over to the next period:
- Excess inventory fully carried over with unit cost $h$.
- Unmet demand fully backlogged with unit cost $b$. 
Sequence of Events

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- The firm makes the following decisions:
  - $x_t - I_t \geq 0$: spot-purchasing, delivered immediately;
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  - $d_t \in [d, \bar{d}]$: expected demand in the consumer market.
Sequence of Events

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Bellman Equation

$V_t(l_t|c_t)$ =the maximal expected discounted profit in periods $t, t - 1, \cdots, 1$
with starting inventory level $l_t$ and cost $c_t$ in period $t$.

Terminal condition: $V_0(l_0|c_0) = 0$. 
Bellman Equation

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Bellman equation:

\[
V_t(l_t|c_t) = c_t l_t + \max_{x_t \geq l_t, q_t \geq 0, d_t \in [d, \bar{d}]} J_t(x_t, q_t, d_t | c_t), \quad \text{where}
\]

\[
J_t(x_t, q_t, d_t | c_t) = -c_t l_t + \mathbb{E}\{p(d_t)D_t - c_t(x_t - l_t) - \gamma c_t q_t - h(x_t - D_t)^+ \\
- b(x_t - D_t)^- + \alpha V_{t-1}(x_t + q_t - D_t | s_t(c_t, \xi_t)) | c_t\}
\]

\[
= R(d_t) - c_t x_t - \gamma c_t q_t + \Lambda(x_t - d_t) + \Psi_t(x_t + q_t - d_t | c_t)
\]

with \( \Lambda(y) = \mathbb{E}\{-h(y - \epsilon_t)^+ - b(y - \epsilon_t)^-\} \),

and \( \Psi_t(y | c_t) = \alpha \mathbb{E}\{V_{t-1}(y - \epsilon_t | s_t(c_t, \xi_t)) | c_t\} \).
Optimal Policy

- \((x_t^*(l_t, c_t), q_t^*(l_t, c_t), d_t^*(l_t, c_t))\): the optimal decisions in period \(t\).
- \(\Delta_t^*(l_t, c_t) := x_t^*(l_t, c_t) - d_t^*(l_t, c_t)\): the optimal safety stock.
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- The cost-dependent order-up-to/pre-order-up-to list-price policy.

- If \( l_t \leq x_t(c_t) \), order from both channels and charge a list price.

- If \( l_t \in (x_t(c_t), l_t^*(c_t)) \), order via the forward-buying contract only and charge a discounted price.

- If \( l_t \geq l_t^*(c_t) \), order nothing.
Impact of Cost Volatility

- Intuition: higher cost volatility $\rightarrow$ lower profit.
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- Actually, the prediction is reversed:

Theorem 1

For two procurement cost processes $\{c_t\}_{t=T}^{1}$ and $\{\hat{c}_t\}_{t=T}^{1}$, assume that for every $t = T, T - 1, \cdots, 1$, $s_t(c_t, \xi_t)$ and $\hat{s}_t(c_t, \xi_t)$ are concavely increasing in $c_t$ for any realization of $\xi_t$. The following statements hold:

(a) For any $l_t$, $V_t(l_t|c_t)$ is convexly decreasing in $c_t$.

(b) If $\{c_t\}_{t=T}^{1}$ and $\{\hat{c}_t\}_{t=T}^{1}$ are identical except that $\hat{s}_t(c_T, \xi_T) \geq_{cx} s_T(c_T, \xi_T)$ for some $c_T$ and $T$, $\hat{V}_t(l_t|c_t) \geq V_t(l_t|c_t)$ for each $(l_t, c_t)$ and $t$, where $\geq_{cx}$ refers to larger in convex order, and $\{\hat{V}_t(l_t|c_t)\}_{t=T}^{1}$ are the value functions associated with $\{\hat{c}_t\}_{t=T}^{1}$.
Impact of Cost Volatility (Cont’d)

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  - Decisions made *posterior* to cost realization in each period.

- Respond to cost volatility.
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- Capacity management and newsvendor network models with responsive/postponed pricing: Van Mieghem and Dada (1999), Chod and Rudi (2005) and Bish et al. (2012).
Impact of Cost Volatility: Assumptions

- Risk neutrality is necessary for Theorem 1 to hold.
  - Opposite predictions in the OM-finance literature: risk aversion.
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Impact of Cost Volatility: Assumptions

- Risk neutrality is necessary for Theorem 1 to hold.
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- The concavity of $s_t(c_t, \xi_t)$ generally can be satisfied (e.g., GBMs, mean-reverting processes).

- When $s_t(c_t, \xi_t)$ is not concave in $c_t$, the result holds for the majority of numerical cases (exceptions may exist when the initial cost is low), in particular when the initial cost follows the stationary distribution.
Optimal Response to Cost Volatility

\[ J_t(x_t, q_t, d_t|c_t) = [R(d_t) - c_t d_t] + [\Lambda(\Delta_t) - (1 - \gamma) c_t \Delta_t] \\ + [\Psi_t(\Delta_t + q_t|c_t) - \gamma c_t(\Delta_t + q_t)]. \]

- Three objectives: (a) generating revenue, (b) hedging against demand uncertainty, and (c) speculating on future costs.
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- Optimal sales price: \( p_t^*(l_t, c_t) \uparrow c_t \). The firm passes (part of) the cost risk to customers.
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- Optimal safety-stock and spot-purchasing: \( \Delta_t(c_t), x_t(c_t) \downarrow c_t \), if \( \gamma \leq 1 \); \( \Delta_t(c_t) \uparrow c_t \), if \( \gamma > 1 \).
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- Optimal forward-buying quantity:
  Generally not monotone in \( c_t \).
Strategic Relationship between Dynamic Pricing and Dual-Sourcing

- Dynamic pricing and dual-sourcing may be either strategic complements or substitutes.
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- **Complements:** if the additional sourcing channel is forward-buying.

- **Substitutes:** if the additional sourcing channel is spot-purchasing.
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- **Rationale**: dynamic pricing mitigates the demand uncertainty risk, but the additional sourcing channel may dampen or intensify the demand uncertainty risk.
Conclusion: Takeaway Insights

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- Dynamic pricing and dual-sourcing may be either complements or substitutes.
  - Dynamic pricing dampens both demand and cost risks, while dual-sourcing may either mitigate or intensify the demand risk.
Thank you!

Questions?