Timing of Effort and Reward: Three-sided Moral Hazard in a Two-Period Model *

Jun Yang†
Washington University in Saint Louis

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Abstract

Business often needs to face the problem of providing incentives for employees to work together effectively on projects that develop over time. This paper derives the optimal contract in an intertemporal model with three-sided moral hazard. The optimal timing of compensation reflects the timing of effort, with compensation for upfront effort preceding compensation for effort over time. Deferring compensation for agents exerting effort over time improves their incentives without impairing incentives for upfront effort. The exact pattern of compensation depends on the relative severity of the agents' incentive problems.

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†**Correspondence:** Jun Yang, John M. Olin School of Business, Washington University in St. Louis, Campus Box 1133, One Brookings Drive, St. Louis, MO 63130-4899. Tel: (314) 477-7969. Fax: (314) 935-6359. Email: yang@olin.wustl.edu.

1 Introduction

Similar to *Timing of Effort and Reward: Three-sided Moral Hazard in a Continuous-Time Model*.

Yang (2003) proposes a continuous-time model in a setting similar to this paper. The optimal incentive contract induces all agents, whose effort choices have different timings, to invest efficiently. The continuous-time model gives a clear cut point at which to switch payments from the entrepreneur to the manager and the chef. In one situation, where the entrepreneur exerts up-front effort and the manager/chef puts in effort over time, there is a critical deadline before which the entrepreneur receives all proceeds of the project; afterwards, the manager claims all cash flow. This critical deadline is determined by the relative severity of the two moral hazard problems. In an integrated model with all three agents, the entrepreneur receives all proceeds of the project before a critical deadline, while the manager and the chef split the proceeds afterwards. In that paper, however, the optimal sharing rule between the manager and the chef in the integrated model is only derived for a special case where the two agents face equally severe moral hazard problems. In that special case, the manager and the chef each claims one half of the proceeds after the critical deadline. This paper, on the other hand, develops a complete solution to the three-sided moral hazard problem in a two-period setting.

The rest of the paper proceeds as follows. Section 2 contains a model with two agents whose ongoing effort symmetrically affects subsequent project failure. Section 3 develops a model with one agent exerting effort at the outset and another agent putting in ongoing effort. Section 4 provides an integrated model that resembles the properties of both models above. Section 5 concludes.

2 Fixed Fraction Contract

In this section, we derive an optimal incentive contract where two agents exert ongoing effort that symmetrically affect the failure rate of a project. In the example of an upscale restaurant, the functions of the manager and the entrepreneur are combined. A manager/entrepreneur, who is labelled the *manager*, operates an upscale restaurant. A chef (the head chef if there are multiple chefs) works in each period to ensure the quality of food served in the restaurant. The ongoing effort choices of the manager and the chef jointly determine the survival rate of the restaurant and in turn its expected lifespan.[1] We need to design an optimal incentive scheme to induce both agents to invest efficiently.

The general formulation of the problem is as follows. An upscale restaurant needs an initial investment I from the agents. The manager chooses the effort levels in two periods $e_1(1)$ and $e_1(2)$ while the chef chooses $e_2(1)$ and $e_2(2)$. (Subscript 1 represents the manager and subscript 2 represents the chef.) All effort levels are normalized to fall in $[0, 1]$ to denote effort intensities. Therefore, the difference between 1 and the effort level of an agent is the shirking of the agent. We assume that the shirking of agents has a cumulative impact on the absolute failure rate of the project. One possible explanation is that shirking increases the probability of failure by destroying items related to the productivity of the project, such as the value of human capital.[2] In particular, in period 1, the manager's shirking $1-e_1(1)$ influences the project failure rates in both periods with a multiplier λ_1 while in period 2, shirking $1 - e_1(2)$ affects the project failure rate with the same multiplier

[1]For notational convenience, the failure rate is used in place of the survival rate.

[2]Indeed, all results remain the same qualitatively if there is a countervailing term in the absolute failure rate (2) that characterizes inertia in the market, for example, $a - 0.5bi^2$ with $b > 0$.

but only in period 2. Similar meanings apply to $1-e_2(1)$ and $1-e_2(2)$, while the influence multiplier is denoted as λ_2.

The absolute failure rates (mass functions of the failure time) of the project in periods 1 and 2 are

$$\begin{aligned} p(1) &= \lambda_1\big(1-e_1(1)\big)+\lambda_2\big(1-e_2(1)\big) \text{ and} \\ p(2) &= \lambda_1\big(1-e_1(1)+1-e_1(2)\big)+\lambda_2\big(1-e_2(1)+1-e_2(2)\big), \end{aligned} \tag{1}$$

respectively. Therefore, the probability that the project fails before periods 1 and 2 is

$$\begin{aligned} F(1) &= p(1)=\lambda_1\big(1-e_1(1)\big)+\lambda_2\big(1-e_2(1)\big) \text{ and} \\ F(2) &= p(1)+p(2)=\lambda_1(3-2e_1(1)-e_1(2))+\lambda_2(3-2e_2(1)-e_2(2)). \end{aligned} \tag{2}$$

It requires $3(\lambda_1+\lambda_2)\leq 1$ to ensure that $F(2)$ falls in $[0,1]$ for all feasible effort choices.

A constant hazard rate is an alternative to describe the influence of shirking on the failure of the project. The hazard rate, which is conditional on survival until t, equals the absolute failure rate divided by the probability of surviving until t. Using hazard rates usually simplifies the stationarity analysis and makes the solution tractable in infinite horizon models. With two periods, however, cross terms created by a constant hazard rate would complicate the algebra in this paper. Therefore, we use the absolute failure rate and assume that it is separable in two sources of moral hazard.

Assuming quadratic costs of effort,[3] we write the expected social surplus as

$$\Pi = y\big(1-F(1)\big)+y\big(1-F(2)\big)-k_1e_1(1)^2-k_1e_1(2)^2-k_2e_2(1)^2-k_2e_2(2)^2-I, \tag{3}$$

[3]The cost of effort could be any function that is strictly convex, differentiable, and increasing with $K(0)=0$. We choose quadratic functions mainly because they simplify the algebra while rendering good economic interpretations.

where $y\big(1-F(1)\big)$ and $y\big(1-F(2)\big)$ are the expected cash flow in periods 1 and 2, respectively; and where k_1 and k_2 are the unit costs of exerting effort for the manager and chef, respectively. The total initial investment I is sunk once the project starts. The initial investment is used for equipment that is specifically geared to the restaurant at that location so the resale value is very low.

To focus on incentives, we assume that both the manager and the chef have linear utilities. Hence, the utility of an agent equals to the expected surplus. As in most agency problems, we assume that the utility of consumption is separable from the disutility of effort.[4]

Each agent faces an incentive compatibility constraint and a participation constraint. [5]The manager chooses effort levels $e_1(1) \in [0,1]$ and $e_1(2) \in [0,1]$ to maximize the expected surplus:

$$\Pi_1 = c_1(1)\big(1-F(1)\big) + c_1(2)\big(1-F(2)\big) - k_1 e_1(1)^2 - k_1 e_1(2)^2 - I_1, \quad (4)$$

where $c_1(1) \in [0,y]$ and $c_1(2) \in [0,y]$ are the payments to the manager in periods 1 and 2, respectively. The remaining cash flow $y-c_1(1)$ and $y-c_1(2)$ go to the chef.

The payments to the agents are bounded by the proceeds of the project. These financial constraints indicate that neither the manager nor the chef have "out-of-pocket" money once the project starts.[6]

Plugging (2) into (4), we obtain the expected surplus of the manager as

$$\Pi_1 = C_1 + \lambda_1\big(c_1(1) + 2c_1(2)\big)e_1(1) + \lambda_1 c_1(2) e_1(2) - k_1 e_1(1)^2 - k_2 e_1(2)^2, \quad (5)$$

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[4]The result remains the same if future cash flow is discounted.

[5]We maximize the expected social surplus which is one way of finding an efficient (second-best) contract. This approach has the merit of focusing our effort on the agency problem. Alternatively, we could maximize the utility of one agent subject to the individual rationality constraint of the other agent to map out a frontier of efficient contracts. Or, we could think of agents as having endowments and entering into a bargaining problem.

[6]This relates to the limited liability of equity claims or nonrecourse debt claims in practice.

where $C_1 \equiv \big(c_1(1)+c_1(2)\big)-(\lambda_1+\lambda_2)\big(c_1(1)+3c_1(2)\big)+\lambda_2\big(c_1(1)+2c_1(2)\big)e_2(1)+\lambda_2 c_1(2)e_2(2)-I_1$ collects terms independent of $e_1(1)$ and $e_1(2)$. Maximizing Π_1 with respect to $e_1(1)$ and $e_1(2)$ generates the optimal effort choices of the manager as functions of payments:

$$\begin{aligned} e_1(1) &= \frac{\lambda_1}{2k_1}\big(c_1(1)+2c_1(2)\big) \\ e_1(2) &= \frac{\lambda_1}{2k_1}c_1(2). \end{aligned} \tag{6}$$

Accordingly, the optimal effort choices of the chef are

$$\begin{aligned} e_2(1) &= \frac{\lambda_2}{2k_2}\big(3y-c_1(1)-2c_1(2)\big) \\ e_2(2) &= \frac{\lambda_2}{2k_2}\big(y-c_1(2)\big). \end{aligned} \tag{7}$$

Due to budget-balancing constraints, the two agents face conflicts of interest: the more powerful the incentive to one agent, the less powerful the incentive to the other agent.[7] Given the two incentive compatibility constraints above, the payments to the manager $c_1(1) \in [0, y]$ and $c_1(2) \in [0, y]$ are chosen to maximize the expected social surplus. Plugging (6) and (7) into (3) and collecting terms, we obtain the maximum expected social surplus

$$\begin{aligned} \Pi =& C + 3s_1 y c_1(1) - \frac{1}{2}\Big(s_1+s_2\Big)c_1(1)^2 + 7s_1 y c_1(2) - \frac{5}{2}\Big(s_1+s_2\Big)c_1(2)^2 \\ &- 2(s_1+s_2)c_1(1)c_1(2), \end{aligned} \tag{8}$$

where $C \equiv 5s_2y^2 + (1-4\lambda_1-4\lambda_2)y - I$ collects terms independent of $c_1(1)$ and $c_1(2)$; Denote $s_1 \equiv \frac{\lambda_1^2}{2k_1}$ and $s_2 \equiv \frac{\lambda_2^2}{2k_2}$ as the severity of the moral hazard problems that the manager and the chef face, respectively. A bigger s_1 corresponds to a bigger influence multiplier λ_1 or a smaller unit cost of exerting

[7] A budget breaker (see Holmström 1982) may help achieve the first-best outcome. However, a budget breaker is infeasible if agents are wealth-constrained. Also, a budget breaker may introduce some other problems to the incentive scheme. For example, the budget breaker may collude with one agent to expropriate the other agent.

effort k_1. Therefore, the bigger the s_1, the more severe the moral hazard problem faced by the manager. [8]

Maximizing Π with respect to $c_1(1)$ and $c_1(2)$ gives the optimal social surplus

$$\Pi^* = \frac{2(s_1^2 + s_1 s_2 + s_2^2)}{s_1 + s_2} y^2 + (2 - 4\lambda_1 - 4\lambda_2) y - I \tag{9}$$

at

$$c_1(1)^* = c_1(2)^* = \frac{s_1}{s_1 + s_2} y. \tag{10}$$

The optimal sharing rule above allocates shares proportional to the relative severity of the two moral hazard problems. In each period the manager receives $\frac{s_1}{s_1+s_2}y$ while the chef receives $\frac{s_2}{s_1+s_2}y$. The more severe the moral hazard problem an agent faces, the more payments the agent receives in each period. This allocation rule gives the agents proper incentives to exert effort.

Notice that the costs of effort are additive convex functions. Hence, given the complementarity of agents' effort, joint provision of effort is value-enhancing.

Plugging (10) into (6) and (7), we obtain the following the optimal effort levels as

$$\begin{aligned} e_1(1)^* &= \frac{3\lambda_1}{2k_1} \frac{s_1}{s_1 + s_2} y \\ e_1(2)^* &= \frac{\lambda_1}{2k_1} \frac{s_1}{s_1 + s_2} y \\ e_2(1)^* &= \frac{3\lambda_2}{2k_2} \frac{s_2}{s_1 + s_2} y \\ e_2(2)^* &= \frac{\lambda_2}{2k_2} \frac{s_2}{s_1 + s_2} y. \end{aligned} \tag{11}$$

[8] The severity of moral hazard measures the difference between the first best effort and the cost-minimizing effort. The first-best solution could be derived by maximizing the expected surplus subject to the participation constraint of each agent. Given that the agents have the right endowments and bargaining powers, the first best effort levels are $e_i(1)^{**} = \frac{3\lambda_i}{2k_i} y$ and $e_i(2)^{**} = \frac{\lambda_i}{2k_i} y$, for $i = 1, 2$.

To ensure $e_1(1)^*, e_1(2)^* \leq 1$, $\frac{\frac{3\lambda_1^3}{4k_1^2}}{\frac{\lambda_1^2}{k_1}+\frac{\lambda_2^2}{k_2}} y \leq 1$ is required and to ensure $e_2(1)^*, e_2(2)^* \leq 1$, $\frac{\frac{3\lambda_2^3}{4k_2^2}}{\frac{\lambda_1^2}{k_1}+\frac{\lambda_2^2}{k_2}} y \leq 1$ is required.

The results are summarized in Proposition 1.

PROPOSITION 1 *Given that parameters* λ_1, λ_2, k_1, k_2, *and* y *satisfy* $3(\lambda_1+\lambda_2) \leq 1$, $\frac{\frac{3\lambda_1^3}{4k_1^2}}{\frac{\lambda_1^2}{k_1}+\frac{\lambda_2^2}{k_2}} y \leq 1$ *and* $\frac{\frac{3\lambda_2^3}{4k_2^2}}{\frac{\lambda_1^2}{k_1}+\frac{\lambda_2^2}{k_2}} y \leq 1$, *two agents which exert ongoing private effort that symmetrically impacts subsequent project failure share the proceeds of the project in proportions* $\frac{s_1}{s_1+s_2}$ *and* $\frac{s_2}{s_1+s_2}$, *where* $s_1 = \frac{\lambda_1^2}{2k_1}$ *and* $s_2 = \frac{\lambda_2^2}{2k_2}$ *denote the severity of the moral hazard problem faced by the manager and the chef, respectively.*

The optimal incentive contract in Proposition 1 relates to equity claims in the real world. Because the manager and the chef both exert ongoing effort that affects the project failure, they each owns a fraction of the restaurant business to be induced to expend efficient effort (second-best). Their shares depend on the relative severity of the two moral hazard problems faced.[9]

3 Deferred Compensation

In Section 2, both the manager/entrepreneur and the chef put in ongoing effort that symmetrically affects the probability of project failure. The optimal contract splits cash flow of the project between the two agents proportional

[9] A fixed-fraction contract (the so-called proportional sharing rule in this paper) could resolve many informational problems. Assuming stage financing with asymmetric information about the project profit, Admati and Pfleiderer (1994) show that a fixed-fraction contract induces the inside investor to make optimal investment decisions. Wilson (1968) investigates the risk-sharing aspect of equity-like claims where the shares are proportional to the degrees of risk aversion. In the framework of Myers and Majluf (1984), Dybvig and Zender (1991) design a fixed-fraction contract to mitigate the subopmital investment problem faced by the manager. Ross (1973) also derives a linear contract.

to the relative severity of the two moral hazard problems. Essentially, this corresponds to equity claims.

In this section, we discuss a situation where the entrepreneur exerts effort $e_0 \in [0, \phi]$ at the outset and the manager/chef, named the *manager*, puts in ongoing effort in each period, $e_1(1), e_1(2) \in [0, 1]$. The optimal contract pays the entrepreneur early on while allocating the proceeds of the project to the manager in later periods. Specifically, the manager splits the proceeds in period 1 with the entrepreneur and receives all cash flow in period 2 if the manager's moral hazard problem is more severe; on the other hand, if the entrepreneur's moral hazard problem is more severe, the entrepreneur receives all proceeds in period 1 and splits the proceeds in period 2 with the manager. In a knife-edge case where the two agents face equally severe moral hazard problems, the entrepreneur receives all proceeds in period 1 while the manager claims everything in period 2.

The absolute failure rates of the project in periods 1 and 2 are

$$\begin{aligned} p(1) &= (\phi - e_0) + \lambda_1\big(1 - e_1(1)\big) \text{ and} \\ p(2) &= (\phi - e_0) + \lambda_1\big(1 - e_1(1) + 1 - e_1(2)\big), \end{aligned} \tag{12}$$

respectively. The probability that the project fails before periods 1 and 2 are

$$\begin{aligned} F(1) &= p(1) = (\phi - e_0) + \lambda_1\big(1 - e_1(1)\big) \text{ and} \\ F(2) &= p(1) + p(2) = 2(\phi - e_0) + \lambda_1\big(3 - 2e_1(1) - e_1(2)\big), \end{aligned} \tag{13}$$

respectively. The expected surplus of the entrepreneur is

$$\Pi_0 = \big(y - c_1(1)\big)\big(1 - F(1)\big) + \big(y - c_1(2)\big)\big(1 - F(2)\big) - \gamma e_0^2 - I_0, \tag{14}$$

where $c_1(1) \in [0, y]$ and $c_1(2) \in [0, y]$ are the payments to the manager in periods 1 and 2, respectively; the remaining proceeds $y - c_1(1)$ and $y - c_1(2)$ go to the entrepreneur. γ is the unit cost of exerting effort for the entrepreneur, and I_0 is the initial investment of the entrepreneur.

Plugging (13) into (14), we have

$$\Pi_0 = C_0 + \big(3y - c_1(1) - 2c_1(2)\big)e_0 - \gamma e_0^2, \tag{15}$$

where $C_0 \equiv \big(2-3\phi-\lambda_1(4-3e_1(1)-e_1(2)\big)y+\lambda_1 c_1(1)\big(1-e_1(1)\big)+\lambda_1 c_1(2)\big(3-2e_1(1)-e_1(2)\big) - I_0$ collects terms independent of e_0.

Maximizing Π_0 with respect to e_0 gives the optimal effort choice of the entrepreneur

$$e_0 = \frac{1}{2\gamma}\big(3y - c_1(1) - 2c_1(2)\big). \tag{16}$$

Accordingly, the optimal effort levels of the manager are

$$\begin{aligned} e_1(1) &= \frac{\lambda_1}{2k_1}\big(c_1(1) + 2c_1(2)\big) \\ e_1(2) &= \frac{\lambda_1}{2k_1}c_1(2). \end{aligned} \tag{17}$$

According to (16) and (17), the entrepreneur and the manager have conflicts of interest due to the budget-balancing constraint. If we reduce $c_1(1)$ by a small amount and increase $c_1(2)$ by half of that amount while keeping both $c_1(1)$ and $c_1(2)$ feasible and $c_1(1) + 2c_1(2)$ constant, then the incentive for the entrepreneur to exert effort upfront is unchanged according to (16). On the other hand, this adjustment keeps the manager's incentive to work hard in period 1 unchanged while better motivating him to exert effort in period 2 according to (17). Therefore, under the optimal allocation rule, the entrepreneur receives payments as early as possible, if the entrepreneur ever has any, while the manager receives payments later on.

The economic intuition is that the effort choice of the entrepreneur is made at the outset and is sunk. Hence, the incentive to exert effort is not affected by the timing of payments as long as $c_1(1)+2c_1(2)$ is a constant. On the other hand, later payments motivate the manager to work hard throughout because the manager needs to keep the project alive in the early stage to receive any payments at all. In the later stage, the manager works hard as the sole claimant of the cash flow.

The payments to the manager $c_1(1) \in [0, y]$ and $c_1(2) \in [0, y]$ are chosen to maximize the expected social surplus

$$\Pi = y\big(1 - F(1)\big) + y\big(1 - F(2)\big) - \gamma e_0^2 - k_1 e_1(1)^2 - k_1 e_1(2)^2 - I. \tag{18}$$

Plugging (13), (16), and (17) into (18) and using notation $s_1 = \frac{\lambda_1^2}{2k_1}$ and $s_2 = \frac{\lambda_2^2}{2k_2}$, we obtain the expected social surplus

$$\begin{aligned}\Pi =& C + 3s_1 y c_1(1) - \Big(\frac{1}{2}s_1 + \frac{1}{4\gamma}\Big)c_1(1)^2 + 7s_1 y c_1(2) - \Big(\frac{5}{2}s_1 + \frac{1}{\gamma}\Big)c_1(2)^2 \\ &- \Big(2s_1 + \frac{1}{\gamma}\Big)c_1(1)c_1(2),\end{aligned} \tag{19}$$

where $C \equiv \frac{9}{4\gamma}y^2 + (2 - 3\phi - 4\lambda_1)y - I$ collects terms independent of $c_1(1)$ and $c_1(2)$. Maximizing Π with respect to $c_1(1)$ and $c_1(2)$ gives the optimal social surplus

$$\Pi^* = \frac{5s_1^2 + \frac{s_1}{4\gamma}}{s_1 + \frac{1}{2\gamma}}y^2 + \frac{9}{4\gamma}y^2 + (2 - 3\phi - 4\lambda_1)y - I \tag{20}$$

at

$$\begin{aligned}c_1(1)^* &= \frac{s_1 - \frac{1}{\gamma}}{s_1 + \frac{1}{2\gamma}}y \\ c_1(2)^* &= y.\end{aligned} \tag{21}$$

The concavity of the expected social surplus function guarantees that the solution in (21) is the global maximum if $s_1 \geq \frac{1}{\gamma}$ ($c_1(1) \geq 0$).

Notice that s_1 and $\frac{1}{\gamma}$ represent the severity of the moral hazard problems that the manager and the entrepreneur face. If the manager's moral hazard problem is relatively severe ($s_1 \geq \frac{1}{\gamma}$), the manager receives more payments. As a result, the two agents split the proceeds in period 1, and the manager receives all proceeds in period 2.

In a continuous model proposed by Yang (2003), the optimal contract pays the entrepreneur all proceeds of the project before a critical deadline t_d that depends on the parameters of the model. After t_d, all proceeds go to the manager to induce him to work hard throughout. In a discrete case, however, except in a knife-edge case, t_d typically falls between periods. If $s_1 > \frac{1}{\gamma}$, the switching point t_d falls between period 0 and period 1.

On the other hand, if $s_1 < \frac{1}{\gamma}$, the entrepreneur faces a more severe moral hazard problem. The switching of payments t_d then occurs between period 1 and period 2: the entrepreneur receives all proceeds in period 1 and splits the proceeds in period 2 with the manager.[10]

Notice that only in a special case, where $s_1 = \frac{1}{\gamma}$, does the switching occur only in period 1, that is, $t_d = 1$. In this case, the entrepreneur receives all proceeds of the project in period 1 while the manager claims everything in period 2.[11]

If the effort of the entrepreneur is extremely costly, that is, if $\frac{1}{\gamma} = 0$, we have $c_1(1)^* = c_1(2)^* = y$ by (21). The manager thus receives all proceeds in each period and the entrepreneur is excluded from the project. On the other hand, if $\frac{1}{\gamma} \to \infty$, then by (23), we have $c_1(1)^* = c_1(2)^* = 0$ which

[10] If $s_1 < \frac{1}{\gamma}$, the maximum expected social surplus is given by

$$\Pi^* = \frac{\frac{49}{2}s_1^2}{5s_1 + \frac{2}{\gamma}}y^2 + \frac{9}{4\gamma}y^2 + (2 - 3\phi - 4\lambda_1)y - I \tag{22}$$

at

$$\begin{aligned} c_1(1)^* &= 0 \\ c_1(2)^* &= \frac{7s_1}{5s_1 + \frac{2}{\gamma}}y. \end{aligned} \tag{23}$$

[11] To ensure $F(2) \leq 1$, $2\phi + 3\lambda_1 \leq 1$ is required; to ensure $e_0^* \leq \phi$, $\frac{\frac{3}{4\gamma^2}}{\frac{\lambda_1^2}{2k_1} + \frac{1}{2\gamma}}y \leq \phi$ is required; and to ensure $e_1(1)^*, e_1(2)^* \leq 1$, $\frac{\frac{3\lambda_1^3}{4k_1^2}}{\frac{\lambda_1^2}{2k_1} + \frac{1}{\gamma}}y \leq 1$ is required.

indicates that the manager is excluded from the project because the effort of the entrepreneur is costless. The results are depicted in Figure 1 and summarized in Proposition 2.

PROPOSITION 2 *Given that parameters ϕ, γ, λ_1, k_1, and y satisfy $2\phi + 3\lambda_1 \leq 1$, $\frac{\frac{3}{4\gamma^2}}{\frac{\lambda_1^2}{2k_1}+\frac{1}{2\gamma}} y \leq \phi$, and $\frac{\frac{3\lambda_1^3}{4k_1^2}}{\frac{\lambda_1^2}{2k_1}+\frac{1}{\gamma}} y \leq 1$, the manager who exerts ongoing effort and the entrepreneur who expends upfront effort split the proceeds in one period but not the other. If the moral hazard problem faced by the manager is more severe, that is, if $s_1 > \frac{1}{\gamma}$, then the manager divides the proceeds in period 1 (receiving $\frac{s_1-\frac{1}{\gamma}}{s_1+\frac{1}{2\gamma}} y$) with the entrepreneur and claims all cash flow in period 2. On the other hand, if the moral hazard problem faced by the entrepreneur is more severe, that is, if $s_1 < \frac{1}{\gamma}$, then the entrepreneur claims all proceeds in period 1 and splits the proceeds in period 2 (receiving $\frac{\frac{2}{\gamma}-2s_1}{5s_1+\frac{2}{\gamma}} y$) with the manager. In a special case where the manager and the entrepreneur face equally severe moral hazard problems, that is, $s_1 = \frac{1}{\gamma}$, the entrepreneur is the sole claimant of the cash flow in period 1 while the manager receives all proceeds in period 2.*

4 Team of Three

The previous two sections each discussed a two-sided moral hazard problem by combining the roles of some agents in a three-sided moral hazard problem. Some basic intuitions on the optimal incentive contracts are provided. If two agents exert effort overtime that symmetrically affect the project failure rate, they split the proceeds of the project proportional to the relative severity of the moral hazard problems. On the other hand, if one agent puts in effort at the outset while the other agent expends ongoing effort, they split the cash flow in one period but not the other, depending on the relative severity of the two moral hazard problems. In addition, the agent who expends upfront effort always receives payments early on.

This section provides an integrated model where all three agents play active roles. The three-agent case captures a lot of generalities that a two-agent case cannot address. In the restaurant example, the entrepreneur exerts effort at the outset in setting up the restaurant while the manager and the chef put in effort in both periods to maintain the quality of the restaurant and to achieve cash flow. The effort choices of the manager and the chef symmetrically affect the failure rate of the business. We need to determine cash flow allocations to induce all three agents to invest efficiently.

The absolute project failure rates (mass functions of the failure time) in periods 1 and 2 are

$$\begin{aligned} p(1) &= (\phi - e_0) + \lambda_1\big(1 - e_1(1)\big) + \lambda_2\big(1 - e_2(1)\big) \text{ and} \\ p(2) &= 2(\phi - e_0) + \lambda_1\big(1 - e_1(1) + 1 - e_1(2)\big) + \lambda_2\big(1 - e_2(1) + 1 - e_2(2)\big), \end{aligned} \tag{24}$$

respectively. The impacts of agents' shirking on the project failure rate are cumulative.

The probabilities that the project fails by periods 1 and 2 are

$$\begin{aligned} F(1) &= p(1) \\ &= (\phi - e_0) + \lambda_1\big(1 - e_1(1)\big) + \lambda_2\big(1 - e_2(1)\big) \text{ and} \\ F(2) &= p(1) + p(2) \\ &= 2(\phi - e_0) + \lambda_1\big(3 - 2e_1(1) - e_1(2)\big) + \lambda_2\big(3 - 2e_2(1) - e_2(2)\big), \end{aligned} \tag{25}$$

respectively. To ensure $F(2) \leq 1$, $2(\phi - e_0) + 3(\lambda_1 + \lambda_2) \leq 1$ is required.

The expected social surplus is

$$\Pi = y\big(1 - F(1)\big) + y\big(1 - F(2)\big) - \gamma e_0^2 - k_1 e_1(1)^2 - k_1 e_1(2)^2 - k_2 e_2(1)^2 - k_2 e_2(2)^2 - I. \tag{26}$$

The incentive compatibility constraint for the entrepreneur is as follows. Choose $e_0 \in [0, \phi]$ to maximize the expected surplus of the entrepreneur

$$\begin{aligned}\Pi_0 &= \big(y - c_1(1) - c_2(1)\big)\big(1 - F(1)\big) + \big(y - c_1(2) - c_2(2)\big)\big(1 - F(2)\big) - \gamma e_0^2 - I_0 \\ &= C_0 + \big(y - c_1(1) - c_2(1)\big)e_0 + 2\big(y - c_1(2) - c_2(2)\big)e_0 - \gamma e_0^2,\end{aligned} \tag{27}$$

where C_0 collects terms independent of e_0. This renders e_0 as a function of the payments $c_1(1), c_1(2), c_2(1)$, and $c_2(2)$:

$$e_0 = \frac{1}{2\gamma}\Big(3y - \big(c_1(1) + c_2(1)\big) - 2\big(c_1(2) + c_2(2)\big)\Big). \tag{28}$$

The incentive compatibility constraint for the manager is as follows. Choose $e_1(1), e_1(2) \in [0, 1]$ to maximize the expected surplus of the manager

$$\begin{aligned}\Pi_1 =& c_1(1)\big(1 - F(1)\big) + c_1(2)\big(1 - F(2)\big) - k_1 e_1(1)^2 - k_1 e_1(2)^2 - I_1 \\ =& C_1 + \lambda_1(c_1(1) + 2c_1(2))e_1(1) - k_1 e_1(1)^2 + \lambda_1 c_1(2)e_1(2) - k_1 e_1(2)^2,\end{aligned} \tag{29}$$

where C_1 collects terms independent of $e_1(1)$ and $e_1(2)$. This renders $e_1(1)$ and $e_1(2)$ as functions of the payments $c_1(1), c_1(2), c_2(1)$, and $c_2(2)$:

$$\begin{aligned}e_1(1) &= \frac{\lambda_1}{2k_1}\big(c_1(1) + 2c_1(2)\big) \\ e_1(2) &= \frac{\lambda_1}{2k_1}c_1(2).\end{aligned} \tag{30}$$

Similarly, the effort choices of the chef are

$$\begin{aligned}e_2(1) &= \frac{\lambda_2}{2k_2}\big(c_2(1) + 2c_2(2)\big) \\ e_2(2) &= \frac{\lambda_2}{2k_2}c_2(2).\end{aligned} \tag{31}$$

Substituting the incentive compatibility constraints (28), (30), and (31) into the expected social surplus (26), we obtain the following transformed optimization problem.

Problem 1 Choose $c_1(1) \geq 0$, $c_1(2) \geq 0$, $c_2(1) \geq 0$, $c_2(2) \geq 0$, $c_1(1) + c_2(1) \leq y$, and $c_1(2) + c_2(2) \leq y$ to maximize

$$
\begin{aligned}
\Pi \equiv\ & \pi_0 \\
& + \left(3s_1 y c_1(1) - \frac{1}{2}s_1 c_1(1)^2 - \frac{1}{4\gamma}c_1(1)^2\right) + \left(7s_1 y c_1(2) - \frac{5}{2}s_1 c_1(2)^2 - \frac{1}{\gamma}c_1(2)^2\right) \\
& + \left(3s_2 y c_2(1) - \frac{1}{2}s_2 c_2(1)^2 - \frac{1}{4\gamma}c_2(1)^2\right) + \left(7s_2 y c_2(2) - \frac{5}{2}s_2 c_2(2)^2 - \frac{1}{\gamma}c_2(2)^2\right) \\
& - \left(2s_1 + \frac{1}{\gamma}\right)c_1(1)c_1(2) - \left(2s_2 + \frac{1}{\gamma}\right)c_2(1)c_2(2) - \frac{1}{2\gamma}c_1(1)c_2(1) \\
& - \frac{1}{\gamma}c_1(1)c_2(2) - \frac{1}{\gamma}c_1(2)c_2(1) - \frac{2}{\gamma}c_1(2)c_2(2),
\end{aligned} \tag{32}
$$

where $\pi_0 \equiv \frac{9}{4\gamma}y^2 + (2 - 3\phi - 4\lambda_1 - 4\lambda_2)y - I$ collects terms independent of $c_1(1)$, $c_1(2)$, $c_2(1)$ and $c_2(2)$; $s_1 = \frac{\lambda_1^2}{2k_1}$, $s_2 = \frac{\lambda_2^2}{2k_2}$, and $1/\gamma$ represent the severity of moral hazard problems faced by the manager, the chef, and the entrepreneur.

The unconstrained global optimum is reached at $c_1(2)^* = c_2(2)^* = y$ which is infeasible due to the violation of the financial constraint $c_1(2) + c_2(2) \leq y$. We then maximize the expected social surplus in (32) by binding some of the six feasibility constraints above.

Notice that the influences of the manager's effort and the chef's effort are symmetric, to simplify the presentation, we assume that the chef always faces a more severe moral hazard problem, that is, $s_1 \leq s_2$. A symmetric solution holds for the case where $s_1 \geq s_2$. The solution is summarized in Case I: $s_1 < \frac{2}{7}s_2$ and Case II: $s_1 \in [\frac{2}{7}s_2, s_2]$. In both cases, denote $\rho = \frac{1}{\gamma}/\frac{s_1 s_2}{s_1+s_2}$ as the relative severity of the moral hazard problem faced by the entrepreneur compared to the moral hazard problems faced together by the manager and the chef.

Case I: $s_1 < \frac{2}{7}s_2$**.**

In this case, the moral hazard problem faced by the manager is much less severe than the moral hazard problem faced by the chef. As a result, for any value of ρ, the manager receives fewer proceeds than the chef in both periods.

If $\rho \leq 2$, the manager and the chef split the proceeds of the project in each period proportional to the relative severity of their moral hazard problems. As ρ increases, the relative severity of the entrepreneur's moral hazard problem increases; as a result, the manager and the chef together surrender proceeds to the entrepreneur. Because the cash flow in period 2 is more important to induce the manger and the chef to exert ongoing effort than it is to induce the entrepreneur to exert upfront effort, for any given ρ, together the manager and the chef surrender more proceeds to the entrepreneur in period 1 than in period 2.

In period 1, the manager and the chef split the proceeds in Case 1.1. From Case 1.2 to Case 1.5 as ρ increases gradually, the manager and the chef together receive fewer payments. Notice that the manager receives non-zero payments only in Cases 1.1 and 1.2 where the entrepreneur's moral hazard problem is the least severe.

In period 2, the manager and the chef split the proceeds in Cases 1.1 through 1.3, the chef claims all proceeds in Case 1.4, while the chef and the entrepreneur split the proceeds in Case 1.5. In each period, the ratio of the chef's share to the manager's share increases monotonically as ρ increases.

On the other hand, as ρ increases, the payments to the entrepreneur gradually increase. If $\rho \leq 2$, the entrepreneur receives no payments in either periods. If $2 < \rho \leq \frac{s_1+s_2}{s_1}$, the entrepreneur receives some proceeds in period 1 but nothing in period 2. If $\rho > \frac{s_1+s_2}{s_1}$, the entrepreneurs receives all proceeds in period 1 and some proceeds in period 2. Notice that as ρ approaches infinity, the entrepreneur receives all proceeds in both periods. The details of the solutions are described as follows.

Case 1.1: $\rho \leq 2$. The optimal payments are[12]

$$\begin{aligned} c_1(1)^* = c_1(2)^* &= \frac{s_1}{s_1+s_2} y \\ c_2(1)^* = c_2(2)^* &= \frac{s_2}{s_1+s_2} y. \end{aligned} \tag{34}$$

The effort of the entrepreneur is so costly that the entrepreneur is completely excluded from the project. The manager and the chef split the proceeds of the project in each period. Their shares are proportional to the relative severity of the two moral hazard problems.

Case 1.2: $2 \leq \rho \leq \frac{2(s_1+3s_2)}{3s_2-s_1}$. The optimal payments are

$$\begin{aligned} c_1(1)^* &= \frac{2\gamma s_1^2 s_2 + 6\gamma s_1 s_2^2 + s_1^2 - 2s_1 s_2 - 3s_2^2}{(s_1+s_2)(2\gamma s_1 s_2 + s_1 + s_2)} y \\ c_2(1)^* &= \frac{6\gamma s_1^2 s_2 + 2\gamma s_1 s_2^2 - 3s_1^2 - 2s_1 s_2 + s_2^2}{(s_1+s_2)(2\gamma s_1 s_2 + s_1 + s_2)} y \\ c_1(2)^* &= \frac{s_1}{s_1+s_2} y \\ c_2(2)^* &= \frac{s_2}{s_1+s_2} y. \end{aligned} \tag{35}$$

In period 1, the entrepreneur picks up some cash flow while the ratio of the chef's share to the manager's share goes up. In period 2, the manager and the chef still split the proceeds proportional to the relative severity of the two moral hazard problems faced.

[12] The expected social surpluses in Cases 1.1 through 1.5 are

$$\begin{aligned} \Pi_{11} &= \pi_0 + \frac{20\gamma(s_1^2 + s_1 s_2 + s_2^2) - 9(s_1+s_2)}{4\gamma(s_1+s_2)} y^2 \\ \Pi_{12} &= \pi_0 + \frac{1}{2} \frac{20\gamma s_1^3 s_2 + 38\gamma s_1^2 s_2^2 + 20\gamma s_1 s_2^3 + 10s_1^3 - 7s_1^2 s_2 - 7s_1 s_2^2 + 10s_2^3}{(s_1+s_2)(2\gamma s_1 s_2 + s_1 + s_2)} y^2 \\ \Pi_{13} &= \pi_0 + \frac{1}{2} \frac{98\gamma s_1^2 s_2 + 100\gamma s_1 s_2^2 + 20\gamma s_2^3 + 49s_1^2 - 79s_1 s_2 + 41s_2^2}{2\gamma s_2^2 + 10\gamma s_1 s_2 + 5s_1 + 5s_2} y^2 \\ \Pi_{14} &= \pi_0 + \frac{20\gamma s_2^2 + s_2}{2(2\gamma s_2 + 1)} y^2 \\ \Pi_{15} &= \pi_0 + \frac{49\gamma s_2^2}{2(5\gamma s_2 + 2)} y^2. \end{aligned} \tag{33}$$

Case 1.3: $\frac{2(s_1+3s_2)}{3s_2-s_1} \leq \rho \leq \frac{14(s_1+s_2)}{6s_2-7s_1}$. The optimal payments are

$$\begin{aligned}
c_1(1)^* &= 0 \\
c_2(1)^* &= \frac{2\gamma s_2^2 + 38\gamma s_1 s_2 - 10s_1 - 10s_2}{2\gamma s_2^2 + 10\gamma s_1 s_2 + 5s_1 + 5s_2} y \\
c_1(2)^* &= \frac{14\gamma s_1 s_2 + 7s_1 - 6s_2}{2\gamma s_2^2 + 10\gamma s_1 s_2 + 5s_1 + 5s_2} y \\
c_2(2)^* &= \frac{2\gamma s_2^2 - 4\gamma s_1 s_2 - 2s_1 + 11s_2}{2\gamma s_2^2 + 10\gamma s_1 s_2 + 5s_1 + 5s_2} y.
\end{aligned} \tag{36}$$

The entrepreneur receives more proceeds, surrendered by the manager and the chef, in period 1 than in the previous two cases. The chef receives fewer payments in period 1 which is compensated by more payments in period 2. The manager receives fewer payments in both periods; indeed, nothing in period 1.

Case 1.4: $\frac{14(s_1+s_2)}{6s_2-7s_1} \leq \rho \leq \frac{s_1+s_2}{s_1}$. The optimal payments are

$$\begin{aligned}
c_1(1)^* &= 0 \\
c_2(1)^* &= \frac{2(\gamma s_2 - 1)}{2\gamma s_2 + 1} y \\
c_1(2)^* &= 0 \\
c_2(2)^* &= y.
\end{aligned} \tag{37}$$

The chef splits the proceeds of the project in period 1 with the entrepreneur and claims all proceeds in period 2. Starting in this case, the manager receives no payments in either periods.

Case 1.5: $\frac{s_1+s_2}{s_1} \leq \rho < \infty$. The optimal payments are

$$\begin{aligned}
c_1(1)^* &= c_2(1)^* = c_1(2)^* = 0 \\
c_2(2)^* &= \frac{7\gamma s_2}{5\gamma s_2 + 2} y.
\end{aligned} \tag{38}$$

Now, the moral hazard problem faced by the entrepreneur is so severe that the entrepreneur receives all proceeds in period 1 while splitting the cash flow in period 2 with the chef.

Note that if $\rho \to \infty$, the optimal sharing rule in case 1.5 is reduced to

$$c_1(1)^* = c_2(1)^* = c_1(2)^* = c_2(2)^* = 0, \tag{39}$$

corresponding to the extreme case where both the manager and the chef are excluded from the project because the effort of the entrepreneur is costless.

The knife-edge case where the entrepreneur receives all proceeds in period 1 but nothing in period 2 occurs only at $\rho = \frac{s_1+s_2}{s_1}$, which is equivalent to $s_2 = \frac{1}{\gamma}$. The optimal payments are $c_1(1)^* = c_2(1)^* = c_1(2)^* = 0$ and $c_2(2)^* = y$. This indicates that the entrepreneur receives all proceeds in period 1 and the chef claims everything in period 2 while the manager is indeed excluded from the project.

The results are depicted in Figure 2.

Case II: $s_1 \in [\frac{2}{7}s_2, s_2]$**.**

In this case, the moral hazard problem that the manager faces remains less severe than the one the chef faces. However, the asymmetry here is not as big as in Case I where $s_1 < \frac{2}{7}s_2$. As ρ increases from Case 2.1 to Case 2.6, the relative severity of the moral hazard problem faced by the entrepreneur compared to the moral hazard problems faced by the manager and the chef becomes greater. As a result, the manager and the chef together receive fewer payments in both periods. In addition, they receive fewer payments in period 1 than in period 2 for any ρ due to the incentives to invest efficiently in period 2. Between the manager and the chef, the ratio of the chef's share to the manager's share in each period increases monotonically as ρ increases.

In period 1, the manager and the chef split the proceeds of the project proportional to the severity of the moral hazard problems in Case 2.1. The three agents divide the proceeds in Case 2.2 while the entrepreneur and the chef split the proceeds in Case 2.3. In Cases 2.4 through 2.6, the entrepreneur claims all proceeds in both periods.

In period 2, the chef splits the proceeds with the manager with an increasing ratio in Cases 2.1 through 2.4. The entrepreneur picks up some cash flow in Case 2.5, and splits the proceeds with the chef in Case 2.6.

Case 2.1: $\rho \leq 2$. This case is precisely the same as Case 1.1. The manager and the chef split the proceeds in each period in a proportion determined by the relative severity of the two moral hazard problems.[13]

Case 2.2: $2 \leq \rho \leq \frac{2(s_1+3s_2)}{3s_2-s_1}$. This case is precisely the same as Case 1.2. All three agents divide the proceeds in period 1 while the manager and the chef split the proceeds in period 2.

Case 2.3: $\frac{2(s_1+3s_2)}{3s_2-s_1} \leq \rho \leq \frac{s_2+19s_1}{s_1}$. The optimal payments in this case and in Case 1.3 are the same while the upper bounds of ρ differ. The entrepreneur picks up more cash flow in period 1 than in Case 2.2. The chef receives less proceeds in period 1 which is compensated by more proceeds in period 2. The manager receives no payments in period 1 in Cases 2.3 through 2.6.

[13] The expected social surpluses in Cases 2.1 through 2.6 are

$$\begin{aligned}
\Pi_{21} &= \pi_0 + \frac{20\gamma(s_1^2 + s_1 s_2 + s_2^2) - 9(s_1 + s_2)}{4\gamma(s_1 + s_2)} y^2 \\
\Pi_{22} &= \pi_0 + \frac{20\gamma s_1^3 s_2 + 38\gamma s_1^2 s_2^2 + 20\gamma s_1 s_2^3 + 10 s_1^3 - 7 s_1^2 s_2 - 7 s_1 s_2^2 + 10 s_2^3}{2(s_1 + s_2)(2\gamma s_1 s_2 + s_1 + s_2)} y^2 \\
\Pi_{23} &= \pi_0 + \frac{98\gamma s_1^2 s_2 + 100\gamma s_1 s_2^2 + 20\gamma s_2^3 + 49 s_1^2 - 79 s_1 s_2 + 41 s_2^2}{2(2\gamma s_2^2 + 10\gamma s_1 s_2 + 5 s_1 + 5 s_2)} y^2 \\
\Pi_{24} &= \pi_0 + \frac{49\gamma s_1^2 + 17\gamma s_1 s_2 + 49\gamma s_2^2 - 10(s_1 + s_2)}{10\gamma(s_1 + s_2)} y^2 \\
\Pi_{25} &= \pi_0 + \frac{49}{10} \frac{5\gamma s_1^2 s_2 + 5\gamma s_1 s_2^2 + 2(s_1 - s_2)^2}{5\gamma s_1 s_2 + 2 s_1 + 2 s_2} y^2 \\
\Pi_{26} &= \pi_0 + \frac{49\gamma s_2^2}{2(5\gamma s_2 + 2)} y^2.
\end{aligned} \tag{40}$$

Case 2.4: $\frac{s_2+19s_1}{s_1} \leq \rho \leq \frac{9}{2}$. The optimal payments are

$$\begin{aligned} c_1(1)^* &= c_2(1)^* = 0 \\ c_1(2)^* &= \frac{7s_1 - 2s_2}{5(s_1+s_2)} y \\ c_2(2)^* &= \frac{7s_2 - 2s_1}{5(s_1+s_2)} y. \end{aligned} \tag{41}$$

This is the knife-edge case where the entrepreneur receives all proceeds in period 1 while the manager and the chef split the proceeds in period 2. Notice that in this case, the ratio of the chef's share to the manager's share is independent of ρ.

Case 2.5: $\frac{9}{2} \leq \rho \leq \frac{5(s_1+s_2)}{2(s_2-s_1)}$. The optimal payments are

$$\begin{aligned} c_1(1)^* &= c_2(1)^* = 0 \\ c_1(2)^* &= \frac{7\gamma s_1 s_2 + \frac{14}{5}(s_1 - s_2)}{5\gamma s_1 s_2 + 2s_1 + 2s_2} y \\ c_2(2)^* &= \frac{7\gamma s_1 s_2 - \frac{14}{5}(s_1 - s_2)}{5\gamma s_1 s_2 + 2s_1 + 2s_2} y. \end{aligned} \tag{42}$$

The entrepreneur receives all proceeds in period 1 while the three agents divide the proceeds in period 2.

Case 2.6: $\frac{5(s_1+s_2)}{2(s_2-s_1)} \leq \rho < \infty$. The optimal payments in this case and Case 1.5 are the same while the lower bounds of ρ differ. Claiming all proceeds in period 1, the entrepreneur splits the proceeds in period 2 with the chef. The manager is excluded from the project.[14]

Note that if $\rho \to \infty$, the solution in Case 2.6 is reduced to

$$c_1(1)^* = c_2(1)^* = c_1(2)^* = c_2(2)^* = 0. \tag{43}$$

[14] Case 2.4 is the knife-edge case where the entrepreneur receives all proceeds in period 1 and the manager and the chef split the proceeds in period 2. Case 2.5 allocates all proceeds to the entrepreneur in period 1 and divides cash flow among the three agents in period 2. These two cases did not exist in Case I because the moral hazard problem of the chef is much more severe than the moral hazard problem of the manager in Case I.

In this extreme case, as the effort of the entrepreneur becomes costless, both the manager and the chef are excluded from the project.[15]

The results are depicted in Figure 3.

Example: $s_1 = s_2 \equiv s$**.**

In this symmetric case, because the moral hazard problems faced by the manager and the chef are equally severe, they receive the same payments in both periods for any ρ. The more severe the entrepreneur's moral hazard problem compared to the manager's and chef's moral hazard problems (a bigger ρ), the later the switching of payments. Notice that Cases 2.3 and 2.6 in which the chef receives more cash flow than the manager are infeasible due to the symmetry here.

In period 1, the manager and the chef equally split the proceeds in Case e.1, divide the proceeds with the entrepreneur in Case e.2 and surrender all proceeds to the entrepreneur in Cases e.3 and e.4. In period 2, the manager and the chef equally split the proceeds in Cases e.1 through e.3 and divide the cash flow with the entrepreneur in Case e.4.

Case e.1: $\rho \leq 2$. The optimal payments are[16]

$$c_1(1)^* = c_2(1)^* = c_1(2)^* = c_2(2)^* = \frac{1}{2}y. \tag{45}$$

[15] The baseline parameters in Figure 1 are set at $y = 1$ and $\frac{s_1}{s_2} = \frac{1}{5}$ while those in Figure 2 are $y = 1$ and $\frac{s_1}{s_2} = \frac{1}{2}$.

[16] The expected social surpluses in cases e.1 through e.4 are

$$\begin{aligned}
\Pi_{e1} &= \pi_0 + \frac{15s}{2}y^2 - \frac{9}{4\gamma}y^2 \\
\Pi_{e2} &= \pi_0 + \frac{39\gamma s^2 + 3s}{4(\gamma s + 1)}y^2 \\
\Pi_{e3} &= \pi_0 + \frac{23}{4}sy^2 - \frac{1}{\gamma}y^2 \\
\Pi_{e4} &= \pi_0 + \frac{49\gamma s^2}{5\gamma s + 4}y^2.
\end{aligned} \tag{44}$$

Case e.2: $2 \leq \rho \leq 4$. The optimal payments are

$$\begin{aligned} c_1(1)^* = c_2(1)^* &= \frac{2\gamma s - 1}{\gamma s + 1} y \\ c_1(2)^* = c_2(2)^* &= \frac{1}{2} y. \end{aligned} \tag{46}$$

Case e.3: $4 \leq \rho \leq \frac{9}{2}$. The optimal payments are

$$\begin{aligned} c_1(1)^* = c_2(1)^* &= 0 \\ c_1(2)^* = c_2(2)^* &= \frac{1}{2} y. \end{aligned} \tag{47}$$

Case e.4: $\frac{9}{2} \leq \rho < \infty$. The optimal payments are

$$\begin{aligned} c_1(1)^* = c_2(1)^* &= 0 \\ c_1(2)^* = c_2(2)^* &= \frac{7\gamma s}{5\gamma s + 4} y. \end{aligned} \tag{48}$$

If $\frac{1}{\gamma} \to \infty$, the optimal sharing rule is

$$c_1(1)^* = c_2(1)^* = c_1(2)^* = c_2(2)^* = 0, \tag{49}$$

and the expected social surplus is π_0. The manager and the chef are both excluded from the project because the effort of the entrepreneur is costless.

The results are depicted in Figure 4.

5 Conclusion

We have proposed an optimal dynamic incentive contract to solve a multi-sided moral hazard problem where the effort choices of all agents collectively determine the project failure rate. In this optimal contract, the timing of payments to the agents reflects the timing of effort choices. The agent with upfront effort receives payments early on and the agents with effort over time claim proceeds in later periods. In addition, the share of payments increases with the relative severity of the moral hazard problem faced by each agent.

The models developed in this paper apply to a broader set of contracting problems in financial markets when the timing of effort choices has an intertemporal impact on the form of the optimal contract. One interpretation of the optimal contract is debt and equity claims. Recall that the proportional sharing rules of cash flow rights in Section 2 relates to an equity contract with shares depending on the relative severity of two moral hazard problems; the optimal incentive contract in Section 3 corresponds to a debt contract with repayments depending on the relative severity of two moral hazard problems. The optimal sharing rule in Section 4 is a mixture of debt and equity where the entrepreneur lends to the manager and the chef who jointly own the restaurant.

With this interpretation in mind, these models can be used as an alternative theory to address the multi-sided moral hazard problems in venture capital finance and strategic alliance contracting. Kaplan and Strömberg (2000) document both debt and equity properties in VC contracts. In these VC contracts, VC's security can automatically convert into common stocks under certain conditions (for example, upon IPO). In the context of strategic alliance contracts, Robinson and Stuart (2002) provide evidence on combined equity and debt financing between pharmaceutical and biotechnology firms.

The second possible interpretation of the optimal contract is stock options.[17] The entrepreneur owns the restaurant initially. The manager and the chef have call options on all of the shares of the restaurant. These options expire at the predetermined deadline t_d and the total strike price is $t_d y$, where y is the constant profit flow of the project. If the project is alive at t_d, the manager and the chef together will exercise these options and their total payoffs at $t \in [t_d, 2]$ are $(t - t_d)y$.

[17] I have benefited from discussion with Kerry Back on this point.

The third possible interpretation of the optimal contract relates to deferred compensation. Recall that in the proposed three-sided moral hazard model, the timing of agents' effort is critical: the entrepreneur expends effort at the outset while the manager and the chef expend effort over time. Given linear utility and no discounting, defer the payments to the manager and the chef induces them to work hard while not changing the incentive of the entrepreneur since the upfront effort is sunk once the project starts. As a result, this allocation rule maximizes the expected social surplus.

The models have some limitations. In this study, we have looked at some two-period models that generate many basic intuitions. However, a two-period model may be misleading in some respects. For example, a two-period model cannot distinguish between the following two allocation rules of cash flow rights simply because there are only two periods. One rule only allocates all proceeds in the last period to an agent while a second rule allocates the agent the proceeds in all periods except the first period. If there were three periods, the first rule, which gives the agent proceeds only in the last period, would only allocate the proceeds in period 3 to the agent. On the other hand, the second rule, which gives the agents proceeds in all but the first period, would allocate the proceeds to the agents in both periods 2 and 3.

The second limitation of the model is related to a more general problem with discrete models. In a discrete model, the switching of payments usually occurs between periods. In a continuous model, in contrast, there is a clear-cut time at which to switch payments. A continuous model is also easier to analyze than an N-period discrete model. In a companion paper, Yang (2003) presents a continuous-time model in a similar multi-sided moral hazard setting.

Incorporating the influence of effort choices on the size of cash flow as well as the failure rate would be an interesting extension of this study. It would also be interesting to balance incentives and risk sharing in optimal contracting in the multi-sided moral hazard framework.

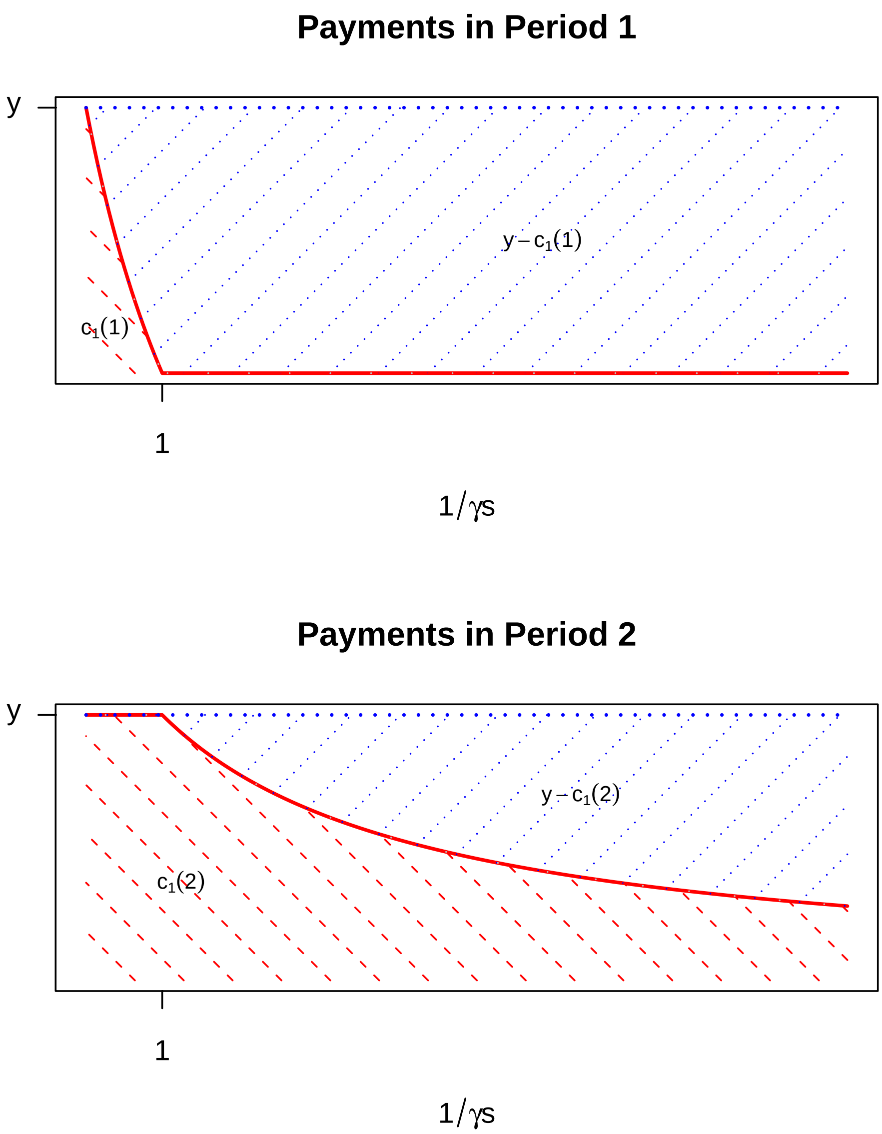
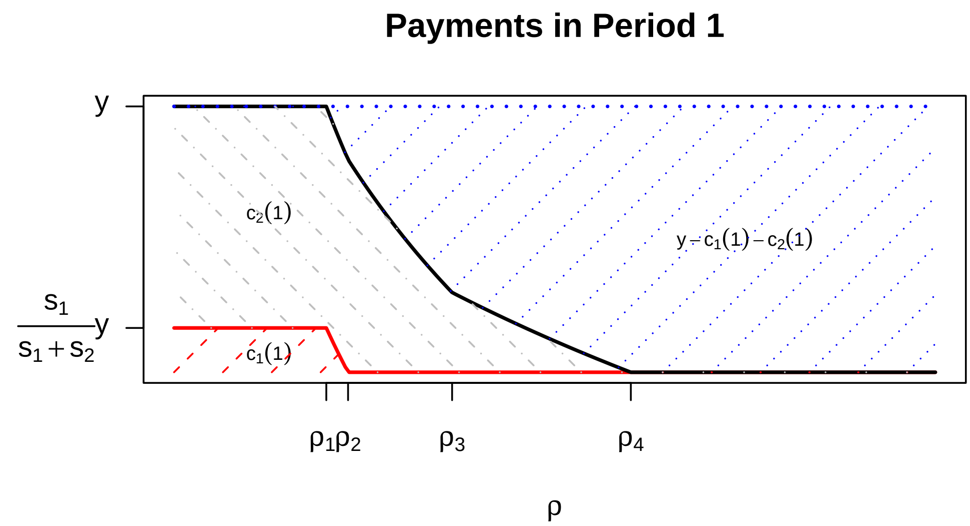


Figure 1: Two-Sided Moral Hazard with Deferred Compensation. The horizontal axes are the relative severity of the entrepreneur's moral hazard, where $\frac{1}{\gamma}$ and s denote the severity of the moral hazard of the entrepreneur and the manager correspondingly. The vertical axis of the top (bottom) panel denotes the payments to the manager $c_1(1)$ ($c_1(2)$) and the entrepreneur $y - c_1(1)$ ($y - c_1(2)$) in period 1 (period 2).



$$\rho_1 = 2, \quad \rho_2 = \frac{2(s_1+3s_2)}{3s_2-s_1}, \quad \rho_3 = \frac{14(s_1+s_2)}{6s_2-7s_1}, \quad \rho_4 = \frac{s_1+s_2}{s_1}$$

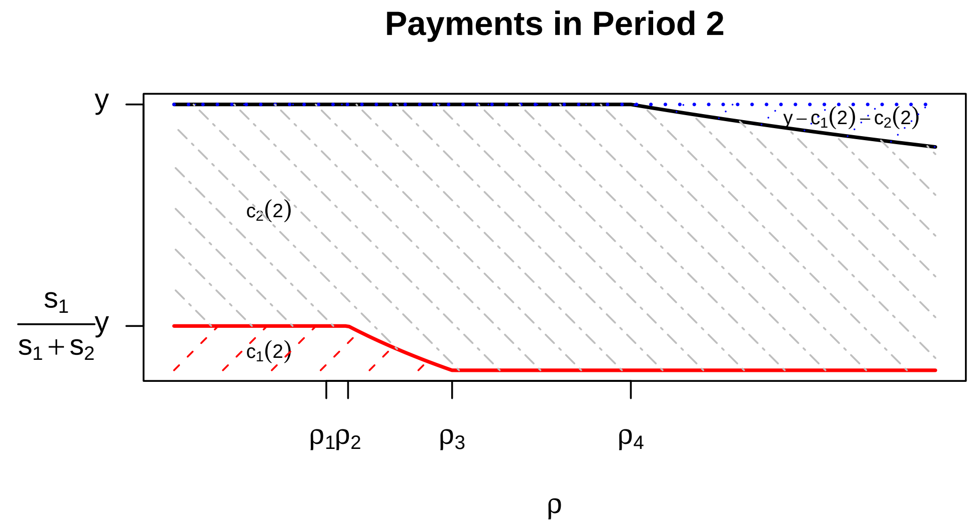
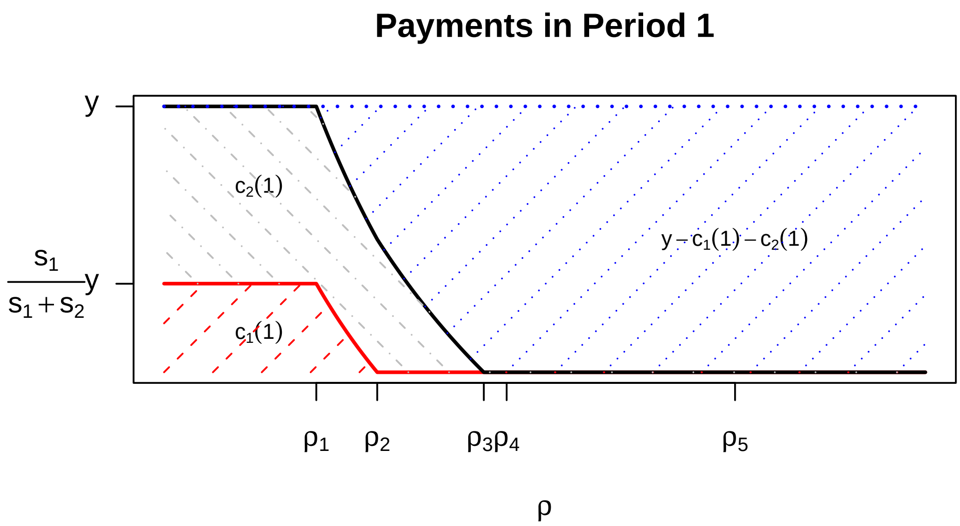


Figure 2: Three-Sided Moral Hazard - Case I, $s_1 < \frac{2}{7}s_2$. The chef's moral hazard much more severe than the manager's. The horizontal axes is the relative severity of the entrepreneur's moral hazard: $\rho = \frac{1}{\gamma}/\frac{s_1 s_2}{s_1+s_2}$. The vertical axis of the top (bottom) panel is the payments to the manager $c_1(1)$ ($c_1(2)$), the chef $c_2(1)$ ($c_2(2)$), and the entrepreneur $y - c_1(1) - c_2(1)$ ($y - c_1(2) - c_2(2)$) in period 1 (period 2). The chef always receives more payments than the manager. Their shares in period 2 are always greater than those in period 1. The payments to the manager and chef both decrease as the entrepreneur's moral hazard becomes more severe while the chef's share decreases more than the manager's.



$\rho_1 = 2$, $\rho_2 = \frac{2(s_1+3s_2)}{3s_2-s_1}$, $\rho_3 = \frac{s_1+19s_2}{5s_1}$, $\rho_4 = \frac{9}{2}$, $\rho_5 = \frac{5(s_1+s_2)}{2(s_2-s_1)}$

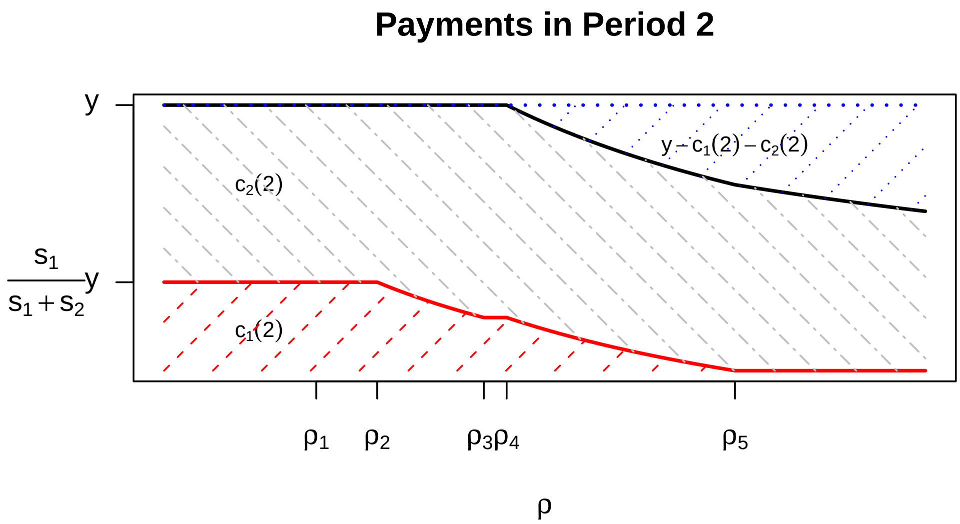


Figure 3: Three-Sided Moral Hazard - Case II, $s_1 \in [\frac{2}{7}s_2, s_2]$. The chef's moral hazard more severe than the manager's. The horizontal axes is the relative severity of the entrepreneur's moral hazard: $\rho = \frac{1}{\gamma}/\frac{s_1 s_2}{s_1+s_2}$. The vertical axis of the top (bottom) panel is the payments to the manager $c_1(1)$ ($c_1(2)$), the chef $c_2(1)$ ($c_2(2)$), and the entrepreneur $y - c_1(1) - c_2(1)$ ($y - c_1(2) - c_2(2)$) in period 1 (period 2). The chef always receives more payments than the manager. Their shares in period 2 are always greater than those in period 1. The payments to the manager and chef both decrease as the entrepreneur's moral hazard becomes more severe while the chef's share decreases faster than the manager's.

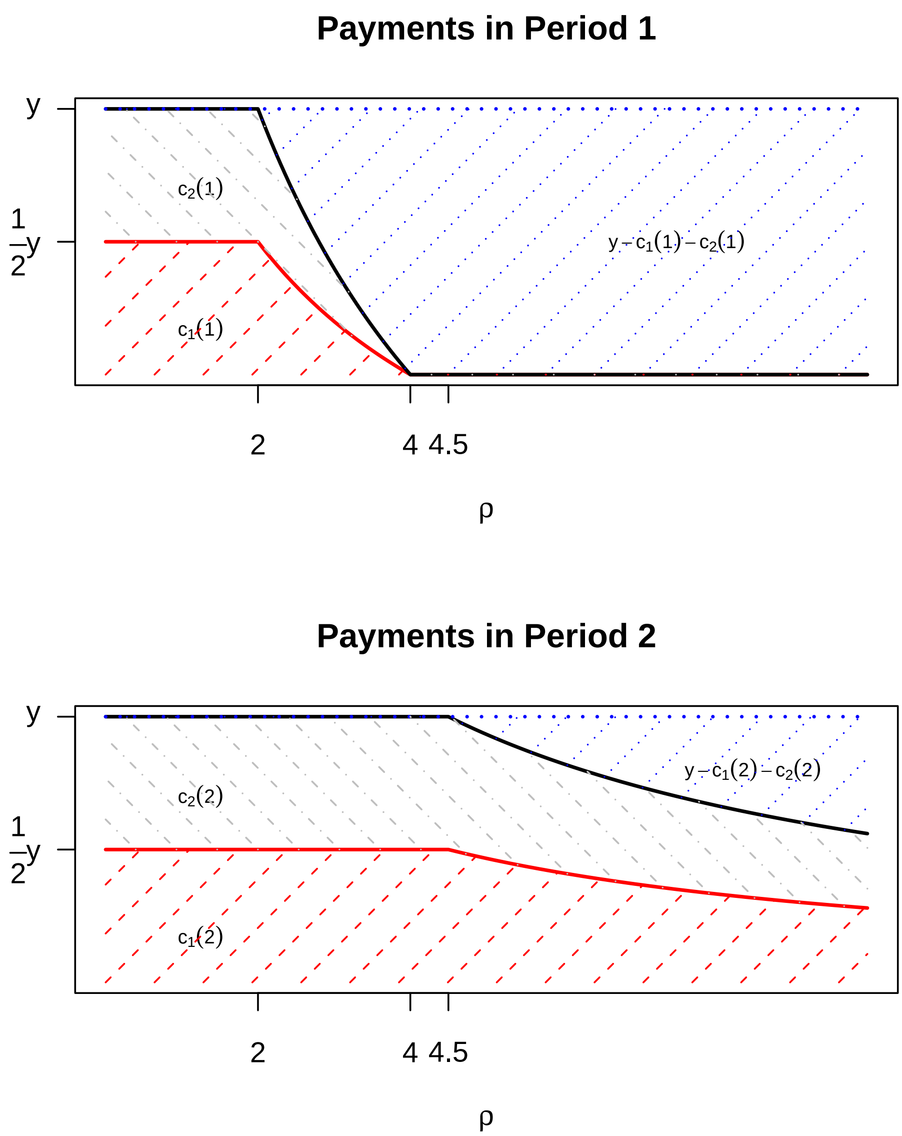


Figure 4: Three-Sided Moral Hazard. The entrepreneur exerts upfront effort while the manager and chef, facing equally severe moral hazard ($s_1 = s_2 \equiv s$), expend effort over two periods. The horizontal axes is the relative severity of the entrepreneur's moral hazard: $\rho = \frac{1}{\gamma} / \frac{s_1 s_2}{s_1 + s_2} = \frac{2}{\gamma s}$. The vertical axis of the top (bottom) panel is the payments to the manager $c_1(1)$ ($c_1(2)$), the chef $c_2(1)$ ($c_2(2)$), and the entrepreneur $y - c_1(1) - c_2(1)$ ($y - c_1(2) - c_2(2)$) in period 1 (period 2).

6 Appendix

Proof of the Three-Agent Case (A Sketch)

The following calculation is similar to the one in the two-agent cases except that the calculation here is much more involved.

The optimal allocation rule of cash flows rights is obtained by maximizing the expected social surplus (32) subject to six feasibility constraints. Choose $c_1(1)$, $c_2(1)$, $c_1(2)$, $c_2(2)$ to maximize

$$\begin{aligned}
&\pi_0 + \big(3s_1 y c_1(1) - \frac{1}{2}s_1 c_1(1)^2 - \frac{1}{4\gamma}c_1(1)^2\big) + \big(7s_1 y c_1(2) - \frac{5}{2}s_1 c_1(2)^2 - \frac{1}{\gamma}c_1(2)^2\big) \\
&\quad + \big(3s_2 y c_2(1) - \frac{1}{2}s_2 c_2(1)^2 - \frac{1}{4\gamma}c_2(1)^2\big) + \big(7s_2 y c_2(2) - \frac{5}{2}s_2 c_2(2)^2 - \frac{1}{\gamma}c_2(2)^2\big) \\
&\quad - \Big(2s_1 + \frac{1}{\gamma}\Big)c_1(1)c_1(2) - \frac{1}{2\gamma}c_1(1)c_2(1) - \frac{1}{\gamma}c_1(1)c_2(2) \\
&\quad - \frac{1}{\gamma}c_1(2)c_2(1) - \frac{2}{\gamma}c_1(2)c_2(2) - \Big(2s_2 + \frac{1}{\gamma}\Big)c_2(1)c_2(2),
\end{aligned}$$

s.t.

$$\begin{aligned}
c_1(1) &\geq 0 \\
c_1(2) &\geq 0 \\
c_2(1) &\geq 0 \\
c_2(2) &\geq 0 \\
c_1(1) + c_2(1) &\leq y \\
c_1(2) + c_2(2) &\leq y,
\end{aligned} \tag{50}$$

where $\pi_0 \equiv \frac{9}{4\gamma}y^2 + (2 - 3\phi - 4\lambda_1 - 4\lambda_2)y - I$ collects terms independent of $c_1(1)$, $c_1(2)$, $c_2(1)$ and $c_2(2)$; $s_1 = \frac{\lambda_1^2}{2k_1}$, $s_2 = \frac{\lambda_2^2}{2k_2}$, and $1/\gamma$ represent the severity of moral hazard problems faced by the manager, the chef, and the entrepreneur.

The global maximum of the unconstrained optimization problem is reached at $c_1(2)^* = c_2(2)^* = y$ which is infeasible due to the violation of the financial constraint $c_1(2) + c_2(2) \leq y$. Corner solutions can be calculated by binding some of the six feasibility constraints above. We then compare the expected social surpluses of these corner solutions and determine the optimal allocation rule in each subinterval of ρ, where $\rho \equiv \frac{1}{\gamma} / \frac{s_1 s_2}{s_1+s_2}$ represents the relative severity of the moral hazard problem faced by the entrepreneur compared to the moral hazard problems faced by the manager and the chef.

Without loss of generality, in all solutions below, we assume that the chef faces a more severe moral hazard problem than the manager, that is, $s_1 \leq s_2$. Given this assumption, in the optimal contract, the payments to the manager are always fewer than the payments to the chef.

Case A.1: $c_1(1) + c_2(1) = y$ and $c_1(2) + c_2(2) = y$.

In this case, the payments are

$$\begin{aligned} c_1(1)^* = c_1(2)^* &= \frac{s_1}{s_1 + s_2} y \\ c_2(1)^* = c_2(2)^* &= \frac{s_2}{s_1 + s_2} y, \end{aligned} \tag{51}$$

and the expected social surplus is

$$\Pi_{A1} = \pi_0 + \frac{5(s_1^2 + s_1 s_2 + s_2^2)}{s_1 + s_2} y^2 - \frac{9}{4\gamma} y^2. \tag{52}$$

Case A.2: $c_1(2) + c_2(2) = y$.

In this case, the payments are

$$\begin{aligned} c_1(1)^* &= \frac{2\gamma s_1^2 s_2 + 6\gamma s_1 s_2^2 + s_1^2 - 2s_1 s_2 - 3s_2^2}{(s_1 + s_2)(2\gamma s_1 s_2 + s_1 + s_2)} y \\ c_2(1)^* &= \frac{6\gamma s_1^2 s_2 + 2\gamma s_1 s_2^2 - 3s_1^2 - 2s_1 s_2 + s_2^2}{(s_1 + s_2)(2\gamma s_1 s_2 + s_1 + s_2)} y \\ c_1(2)^* &= \frac{s_1}{s_1 + s_2} y \\ c_2(2)^* &= \frac{s_2}{s_1 + s_2} y, \end{aligned} \tag{53}$$

and the expected social surplus is

$$\Pi_{A2} = \pi_0 + \frac{20\gamma s_1^3 s_2 + 38\gamma s_1^2 s_2^2 + 20\gamma s_1 s_2^3 + 10s_1^3 - 7s_1^2 s_2 - 7s_1 s_2^2 + 10s_2^3}{2(s_1 + s_2)(2\gamma s_1 s_2 + s_1 + s_2)} y^2. \tag{54}$$

In this solution, constraints $c_1(2)^* \geq 0$, $c_2(2)^* \geq 0$, and $c_1(2)^* + c_2(2)^* \leq y$ are satisfied. Other feasibility constraints determine the range of ρ in which this allocation rule is feasible.

$$\begin{aligned}
& c_1(1)^* \geq 0 \Rightarrow \rho \leq \frac{2(s_1+3s_2)}{3s_2-s_1} \\
& c_2(1)^* \geq 0 \Rightarrow \rho \leq \frac{2(s_2+3s_1)}{3s_1-s_2} \text{ if } s_1 \geq \frac{1}{3}s_2 \\
& c_1(1)^* + c_2(1)^* \leq y \Rightarrow \rho \geq 2.
\end{aligned} \tag{55}$$

Because $\frac{2(s_1+3s_2)}{3s_2-s_1} \leq \frac{2(s_2+3s_1)}{3s_1-s_2}$ given $s_1 \leq s_2$, the feasible range of ρ is $[2, \frac{2(s_1+3s_2)}{3s_2-s_1}]$.

Case A.2.1: $c_1(2) = 0$ and $c_2(2) = y$.

The expected social surplus in this case is smaller than the one in Case A.2. However, the feasible range of ρ in this case is greater than the feasible range of ρ in Case A.2.

The payments are

$$\begin{aligned}
c_1(1)^* &= \frac{6\gamma s_1 s_2 + 3s_1 - 3s_2}{2\gamma s_1 s_2 + s_1 + s_2} y \\
c_2(1)^* &= \frac{2\gamma s_1 s_2 - 5s_1 + s_2}{2\gamma s_1 s_2 + s_1 + s_2} y \\
c_1(2)^* &= 0 \\
c_2(2)^* &= y,
\end{aligned} \tag{56}$$

and the expected social surplus is

$$\Pi_{A21} = \pi_0 + \frac{18\gamma s_1^2 s_2 + 20\gamma s_1 s_2^2 + 9s_1^2 - 17s_1 s_2 + 10s_2^2}{2\gamma s_1 s_2 + s_1 + s_2} y^2. \tag{57}$$

In this solution, constraints $c_1(2)^* \geq 0$, $c_2(2)^* \geq 0$, and $c_1(2)^* + c_2(2)^* \leq y$ are satisfied. Other feasibility constraints determine the range of ρ in which this allocation rule is feasible.

$$\begin{aligned} c_1(1)^* \geq 0 &\Rightarrow \rho \leq \frac{2(s_1+s_2)}{s_2-s_1} \\ c_2(1)^* \geq 0 &\Rightarrow \rho \leq \frac{2(s_1+s_2)}{5s_1-s_2} \text{ if } s_1 \geq \frac{1}{5}s_2 \\ c_1(1)^* \leq y &\Rightarrow \rho \geq \frac{2(s_1+s_2)}{2s_2-s_1} \\ c_1(1)^* + c_2(1)^* \leq y &\Rightarrow \rho \geq 2. \end{aligned} \tag{58}$$

Next, we determine the upper bound and the lower bound of ρ. If $s_1 \leq \frac{1}{3}s_2$, then $\frac{2(s_1+s_2)}{s_2-s_1} \leq \frac{2(s_1+s_2)}{5s_1-s_2}$. Hence, the upper bound of ρ is $\frac{2(s_1+s_2)}{s_2-s_1}$ if $s_1 \leq \frac{1}{3}s_2$ and $\frac{2(s_1+s_2)}{5s_1-s_2}$ if $s_1 \geq \frac{1}{3}s_2$. On the other hand, if $s_1 \leq \frac{1}{2}s_2$, then $2 \geq \frac{2(s_1+s_2)}{2s_2-s_1}$. Hence, the lower bound of ρ is 2 if $s_1 \leq \frac{1}{2}s_2$ and $\frac{2(s_1+s_2)}{2s_2-s_1}$ if $s_1 \geq \frac{1}{2}s_2$. In addition, the lower bound of ρ needs to be smaller than the upper bound. Notice that $2 \leq \frac{2(s_1+s_2)}{s_2-s_1}$ always holds and $2 \leq \frac{2(s_1+s_2)}{5s_1-s_2}$ holds if $s_1 \leq \frac{1}{2}s_2$. However, if $s_1 \geq \frac{1}{2}s_2$, the lower bound $\frac{2(s_1+s_2)}{2s_2-s_1}$ is greater than the corresponding upper bound $\frac{2(s_1+s_2)}{5s_1-s_2}$. Therefore, the solution in Case A.2.1 is feasible only if $s_1 \leq \frac{1}{2}s_2$. The feasible range of ρ for this solution is $[2, \rho^*_{A21}]$ where the upper bound is defined as $\rho^*_{A21} = \frac{2(s_1+s_2)}{s_2-s_1}$ if $s_1 \leq \frac{1}{3}s_2$ and $\rho^*_{A21} = \frac{2(s_1+s_2)}{5s_1-s_2}$ if $s_1 \in [\frac{1}{3}s_2, \frac{1}{2}s_2]$.

Case A.2.2: $c_1(1) = 0$ and $c_1(2) + c_2(2) = y$.

The expected social surplus in this case is smaller than the one in Case A.2 if both solutions are feasible. However, the feasible range of ρ in this case is greater than the feasible range of ρ in Case A.2.

The payments are

$$\begin{aligned} c_1(1)^* &= 0 \\ c_2(1)^* &= \frac{2\gamma s_2^2 + 38\gamma s_1 s_2 - 10s_1 - 10s_2}{2\gamma s_2^2 + 10\gamma s_1 s_2 + 5s_1 + 5s_2} y \\ c_1(2)^* &= \frac{14\gamma s_1 s_2 + 7s_1 - 6s_2}{2\gamma s_2^2 + 10\gamma s_1 s_2 + 5s_1 + 5s_2} y \\ c_2(2)^* &= \frac{2\gamma s_2^2 - 4\gamma s_1 s_2 - 2s_1 + 11s_2}{2\gamma s_2^2 + 10\gamma s_1 s_2 + 5s_1 + 5s_2} y, \end{aligned} \tag{59}$$

and the expected social surplus is

$$\Pi_{A22} = \pi_0 + \frac{98\gamma s_1^2 s_2 + 100\gamma s_1 s_2^2 + 20\gamma s_2^3 + 49 s_1^2 - 79 s_1 s_2 + 41 s_2^2}{2(2\gamma s_2^2 + 10\gamma s_1 s_2 + 5 s_1 + 5 s_2)} y^2. \quad (60)$$

By the same token as in Cases A.2 and A.2.1, the feasible range of ρ for this solution is $[\frac{28}{15}, \rho^*_{A22}]$ where $\rho^*_{A22} = \frac{14(s_1+s_2)}{6s_2-7s_1}$ if $s_1 \leq \frac{2}{7}s_2$ and $\rho^*_{A22} = \frac{s_2+19s_1}{5s_1}$ if $s_1 \in [\frac{2}{7}s_2, s_2]$.

Case A.2.3: $c_1(1) = 0$, $c_2(1) = 0$, and $c_2(2) = y$.

The expected social surplus in this case is smaller than the expected social surpluses in Cases A.2.1 and A.2.2. However, the feasible range of ρ in this case is greater than those in Cases A.2.1 and A.2.2.

The payments are

$$\begin{aligned} c_1(1)^* &= 0 \\ c_2(1)^* &= \frac{2(\gamma s_2 - 1)}{2\gamma s_2 + 1} y \\ c_1(2)^* &= 0 \\ c_2(2)^* &= y, \end{aligned} \quad (61)$$

and the expected social surplus is

$$\Pi_{A23} = \pi_0 + \frac{(20\gamma s_2 + 1)s_2}{2(2\gamma s_2 + 1)} y^2. \quad (62)$$

The feasible range of ρ for this solution is $\rho \leq \frac{s_1+s_2}{s_1}$.

Case A.3: $c_1(1) = 0$, $c_2(1) = 0$, and $c_1(2) + c_2(2) = y$.

The payments are

$$\begin{aligned} c_1(1)^* &= c_2(1)^* = 0 \\ c_1(2)^* &= \frac{7s_1 - 2s_2}{5(s_1 + s_2)} y \\ c_2(2)^* &= \frac{7s_2 - 2s_1}{5(s_1 + s_2)} y, \end{aligned} \quad (63)$$

and the expected social surplus is

$$\Pi_{A3} = \pi_0 + \frac{49\gamma s_1^2 + 17\gamma s_1 s_2 + 49\gamma s_2^2 - 10(s_1 + s_2)}{10\gamma(s_1 + s_2)} y^2. \quad (64)$$

This solution is feasible for any ρ if $s_1 \in [\frac{2}{7}s_2, s_2]$.

Case A.4: $c_1(1) = 0$ and $c_2(1) = 0$.

The payments are

$$\begin{aligned} c_1(1)^* &= c_2(1)^* = 0 \\ c_1(2)^* &= \frac{7\gamma s_1 s_2 + \frac{14}{5}(s_1 - s_2)}{5\gamma s_1 s_2 + 2s_1 + 2s_2} y \\ c_2(2)^* &= \frac{7\gamma s_1 s_2 - \frac{14}{5}(s_1 - s_2)}{5\gamma s_1 s_2 + 2s_1 + 2s_2} y, \end{aligned} \tag{65}$$

and the expected social surplus is

$$\Pi_{A4} = \pi_0 + \frac{49}{10}\frac{5\gamma s_1^2 s_2 + 5\gamma s_1 s_2^2 + 2(s_1 - s_2)^2}{5\gamma s_1 s_2 + 2s_1 + 2s_2} y^2. \tag{66}$$

The feasible range of ρ for this solution is $[\frac{9}{2}, \frac{5(s_1+s_2)}{2(s_2-s_1)}]$ if $s_1 \in [\frac{2}{7}s_2, s_2]$.

Case A.4.1: $c_1(1) = 0$, $c_2(1) = 0$, and $c_1(2) = 0$.

Notice that the expected social surplus in this case is smaller than the one in Case A.4 if both solutions are feasible. However, the feasible range of ρ in this case is greater than that in Case A.4.

The payments are

$$\begin{aligned} c_1(1)^* &= c_2(1)^* = c_1(2)^* = 0 \\ c_2(2)^* &= \frac{7\gamma s_2}{5\gamma s_2 + 2} y, \end{aligned} \tag{67}$$

and the expected social surplus is

$$\Pi_{A41} = \pi_0 + \frac{49\gamma s_2^2}{2(5\gamma s_2 + 2)} y^2. \tag{68}$$

The feasible range of ρ for this solution is $\rho \geq \frac{s_1+s_2}{s_1}$.

Case A.5: $c_1(1) = 0$, $c_2(1) = 0$, $c_1(2) = 0$, and $c_2(2) = 0$.

The expected social surplus is π_0.

This solution is feasible for all possible values of ρ, s_1, and s_2. However, it is dominated by any feasible solutions in Cases A.1 through A.4.1.

Notice that the expected social surplus in Case A.1 is smaller than in Case A.2; and the expected social surplus in Case A.3 is smaller than in Cases A.2 and A.4. However, the solutions in Cases A.1 and A.3 are feasible for wider ranges of ρ than the solutions in Cases A.2 and A.4.

For any feasible value of ρ, the values of the expected social surpluses in Cases A.1 through A.4.1 are compared, two cases then emerge naturally. The solutions are summarized in Case I if $s_1 \leq \frac{2}{7}s_2$ and in Case II if $s_1 \in [\frac{2}{7}s_2, s_2]$; see the text in Section 4.

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