Timing of Effort and Reward: Three-sided Moral Hazard in a Continuous-Time Model [*]

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Abstract

Businesses often need to face the problem of providing incentives for employees to work together effectively on projects that develop over time. This paper derives the optimal contract in an intertemporal model with three-sided moral hazard. The optimal timing of compensation reflects the timing of effort with compensation for up-front effort preceding compensation for effort over time. Deferring compensation for agents exerting effort over time improves their incentives without impairing incentives for up-front effort. The exact pattern of compensation depends on the relative severity of the agents' incentive problems.

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1 Introduction

Businesses often need to face the problem of providing incentives for employees to work together effectively on projects that develop over time. A key feature of many contracting problems with multiple agents is that agents exert effort at different times. This is particularly true for entrepreneurial firms where entrepreneurs set up firms at the outset while managers and workers operate the business over time. This paper derives the optimal contract in an intertemporal model with three-sided moral hazard. The optimal timing of compensation reflects the timing of effort with compensation for up-front effort preceding compensation for continuous effort. Deferring compensation for agents exerting effort over time improves their incentives without impairing incentives for up-front effort because the effort is sunk once the firm is set up. This paper complements static multi-agent models such as Wilson (1968), Holmström (1982) and Bhattacharya and Lafontaine (1995) with dynamics and dynamic single-agent models such as Holmström and Milgrom (1987) and Schättler and Sung (1993) with three-sided moral hazard.

For concreteness, in the remainder of this paper, we will use a stylized restaurant example with an entrepreneur, a chef, and a manager. The entrepreneur has unique expertise in starting restaurants: selecting locations, purchasing equipment, designing menus, decorating restaurants, and hiring a chef and a manager. The chef has special skills in cooking and the manager organizes services in the restaurant: hiring people, purchasing supplies, advertising, and managing all other activities that keep things running smoothly. Differing in expertise, the three agents exert effort at different times: the entrepreneur provides effort at the outset when setting up the restaurant while the chef and the manager exert effort over time to achieve cash flows.

Effort choices of the three agents collectively determine the quality of the restaurant and thereby, affect the probability of business failure and the expected cash flows. How cash flows are allocated over time determines the effort choices of the agents. For a given present value of compensation, the incentives of the entrepreneur are not affected by the timing of payments because his effort is sunk at the outset. In contrast, the manager and the chef would stop working once their payments stop. Therefore, in the optimal contract, the manager and the chef do not receive any payments prior to a known critical date and they split all the cash flows afterwards. Deferring their compensation induces the manager and the chef to work hard throughout. Prior to this critical date, these two agents work hard to keep the project alive so that they can receive payments later on. After this critical date, these agents work hard because they bear the full variation in cash flows. This critical date and the exact pattern of how the manager and the chef divide cash flows depend on the relative severity of their moral hazard problems.

This paper analyzes three models. The general one has three-sided moral hazard as we have been describing. We also look at two models with double-sided moral hazard. Absent the entrepreneur, the manager and the chef both provide ongoing effort that affects the failure rate of the project, and the optimal compensation splits the cash flows of the project in fixed proportions depending on the relative severity of the agents' moral hazard problems. Absent the chef, the entrepreneur provides effort at the outset when setting up the restaurant while the manager (who also works as a chef) exerts effort over time to run the business. The optimal compensation allocates all the cash flows to the entrepreneur prior to a critical date and all the cash flows to the manager afterwards.

In the full model with three-sided moral hazard, the optimal contract is characterized by a critical date prior to which the entrepreneur obtains all proceeds of the restaurant and after which the manager and the chef jointly bear the full variation in cash flows. In particular, the manager and the chef split cash flows equally if their moral hazard problems are equally severe.

Related Literature Early research on principal-agent or multi-agent problems focuses on the optimal design of incentive contracts in a single-period (static) setting. Ross (1973) solves an agency problem between a passive principal and an agent by trading off efficient production and risk sharing. A linear sharing rule is obtained in the optimal contract. Wilson (1968) investigates the risk-sharing aspect of equity-like claims among multiple agents that all make contributions to the production of the enterprize. Their shares are proportional to the degrees of risk aversion. Also in a multi-agent setting, Holmström (1982) shows that the first-best outcome cannot be reached if the principal also exerts effort. Bhattacharya and Lafontaine (1995) summarize the theory and evidence on two-sided moral hazard problems.[1]

In a principal-agent model with a limited-liability constraint, Innes (1990) shows that a debt contract is optimal if the investor's payoff is restricted to be monotonically nondecreasing in firm profits. In general, a "live-or-die" payoff function is optimal: the investor receives nothing if the payoff is higher than a certain threshold and claims all the payoff otherwise. Having much of the same flavor as Innes', our model can be interpreted as an extension of Innes' limited-liability model to a multiple-agent case in continuous time.

A growing literature on continuous-time (dynamic) agency theory examines intertemporal incentive provisions. In a setting when the agent's action affects the drift of the Brownian Motion of the output process, Holmström and Milgrom (1987) show that the optimal contract is linear in output if the agent has an exponential utility. Schättler and Sung (1993) provide a more general mathematical framework of analyzing dynamic agency problems. With exponential utility, the linearity result sustains if the action of the agent affects the diffusion term as well as the drift of the Brownian Motion; see Sung (1995) and Ou-Yang (2003).[2] Lazear (1979) proposes long-term implicit labor contracts under which a younger worker is underpaid while an older worker is overpaid relative to the marginal product. This deferred compensation induces workers to exert effort because a shirking worker will be fired and the promised wage stream is forfeited.[3] Notice that all the continuous-time models above have a passive principal and a single agent while this paper addresses a multi-sided moral hazard problem in continuous

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[1] Two-sided moral hazard problems in a static setting have also been studied in VC contracting; see for example, Repullo and Suarez (1999), Casamatta (2003), and Schmidt (2003).

[2] DeMarzo and Sannikov (2004) propose a continuous-time principal-agent model with privately observable cash flows. The principal may terminate the project and the agent consumes the proceeds while the project is running. The optimal contract resembles a mixture of common equity, a credit line, and a long-term debt or a cash position.

[3] According to Professor Lazear, the appendix that contains the formal proof is no longer available.

time.

The general approach of setting up models in this paper is similar to that of Dybvig and Lutz (1993). However, the structure of the model and its implications differ. In the context of product warranties, Dybvig and Lutz only consider a single case where the producer exerts effort early on during production and the consumer puts in effort in later stages to maintain the product. The optimal contract is a block warranty that transfers wealth from the producer to the consumer in the event of product breakdown in its early life. This study, in contrast, solves moral hazard problems that involve either two agents (symmetric or asymmetric) or three agents. Additionally, this study differs from Dybvig and Lutz (1993) even in the case with two asymmetric agents. In their paper, the consumer pays up-front in exchange for ongoing service benefits of the product. If the product fails in its early life, the producer pays the consumer out of pocket as compensation. In this study, however, all agents are protected by limited liability. As a result, they exclusively share the cash flows generated by the project over time as dividends or coupons.

In contrast to this paper that solves the three-sided moral hazard problem in a special symmetric case, Yang (2004) provides a complete solution to the three-sided moral hazard problem in a two-period setting. However, a two-period model might be misleading in a dynamic context.[4] Furthermore, in a discrete model, the switch of payments usually occurs between periods so it is difficult to implement the compensation scheme. In a continuous model, in contrast, there is a clear-cut time at which to switch payments. Moreover, it is easier to analyze and to interpret comparative statics in a continuous model than in an N-period discrete model.

The rest of the paper proceeds as follows. Section 2 develops a fixed-proportion contract in a two-sided moral hazard model where both agents expend ongoing effort. Section 3 proposes deferred compensation in a two-sided moral hazard model where the agents exert effort at

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[4] For example, a two-period model cannot distinguish between the following two incentive contracts, which differ in general, simply because there are only two periods: under one contract, the agent receives payments only in the last period while under the other contract, the agent receives payments in all but the first period.

differential times. Section 4 solves a three-sided moral hazard problem and Section 6 concludes.

2 Fixed-Proportion Contract

Absent the entrepreneur, the two-sided moral hazard model in this section has the manager and the chef exerting continuous effort (costly and unobservable) that jointly affects the subsequent project failure. The result also holds for an N-sided moral hazard problem ($N \geq 2$) where each agent exerts ongoing effort.

The technical setting is as follows. The manager and the chef invest in a restaurant that has a finite potential lifespan, T.[5] At the outset, the two agents sign a compensation contract that divides the profit flow of the restaurant, b, once it becomes available.[6] Let $c(t)$ denote the payment to the manager and $b - c(t)$ denote the payment to the chef at time t. We assume that $0 \leq c(t) \leq b$ for any $t \in [0, T]$. Essentially, agents' liability is limited to their investment in the project and they cannot get paid more than the available profits.[7] Under this contract, the manager and the chef choose effort plans, $e_1(t)$ and $e_2(t)$ (measurable, normalized to fall in $[0, 1]$ for $t \in [0, T]$), to maintain the quality of the restaurant and thereby, to achieve cash flows.

The formal game has two stages.

- In the first stage, the manager and the chef invest in a restaurant and sign an incentive contract that determines how to split the proceeds of the restaurant over time. The agents then choose their effort plans.[8]
- In the second stage, the manager and the chef expend effort, followed by division of the project cash flows once they are available.

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[5] The initial investment is geared to the unique style of the restaurant at a particular location. Consequently, when the project fails, to the extreme, there is no salvage value of the initial investment.

[6] We use cash flows, profit flows, and proceeds interchangeably.

[7] If instead one agent deposits his expected payments in an account at the outset and this deposit is forfeited upon project failure, then the two-sided moral hazard problem reduces to a one-sided moral hazard problem.

[8] Agents choose effort plans at the outset because they cannot update their beliefs over time without observing other agents' effort.

To make the problem tractable while providing useful insights into dynamic incentive problems with multiple agents, we assume a simple information structure with no intermediate information flow and a constant cash flow rate while the project is running. The only uncertainty here is the timing of project failure. The results are more general than single-period models and are simpler than most dynamic models because there is only one state variable: how long the project has survived.[9]

The absolute failure rate of the project at time s (probability density of the failure time) is assumed to be separable in the two sources of moral hazard:[10]

$$\begin{aligned} f(s;e_1(\cdot),e_2(\cdot);\lambda_1,\lambda_2) &= \lambda_1 \int_{\tau=0}^{s} \big(1-e_1(\tau)\big)d\tau + \lambda_2 \int_{\tau=0}^{s} \big(1-e_2(\tau)\big)d\tau \\ &= (\lambda_1+\lambda_2)s - \lambda_1 \int_{\tau=0}^{s} e_1(\tau)d\tau - \lambda_2 \int_{\tau=0}^{s} e_2(\tau)d\tau. \end{aligned} \tag{1}$$

The effort choice of agent i at time τ, $e_i(\tau)$, $i=1,2$, (normalized to fall in $[0,1]$ for $\tau \in [0,T]$) influences the failure rate for all subsequent times by $\lambda_i(1-e_i(\tau))$. One possible explanation is that shirking increases the probability of failure by destroying items related to the productivity of the project, such as the value of human capital.[11]

The probability that the project fails before t is given by

$$\begin{aligned} F(t;e_1(\cdot),e_2(\cdot);\lambda_1,\lambda_2) &= \int_{s=0}^{t} f(s;e_1(\cdot),e_2(\cdot);\lambda_1,\lambda_2)ds \\ &= \frac{1}{2}(\lambda_1+\lambda_2)t^2 - \lambda_1 \int_{s=0}^{t}\int_{\tau=0}^{s} e_1(\tau)d\tau ds - \lambda_2 \int_{s=0}^{t}\int_{\tau=0}^{s} e_2(\tau)d\tau ds. \end{aligned} \tag{2}$$

In the rest of the paper, the notation will suppress the dependence of the failure probability on $e_1(\cdot)$, $e_2(\cdot)$, λ_1, and λ_2: $f(s)$ is short for $f(s;e_1(\cdot),e_2(\cdot);\lambda_1,\lambda_2)$ and $F(t)$ for $F(t;e_1(\cdot),e_2(\cdot);\lambda_1,\lambda_2)$. Thus, the probability that the project fails before reaching its maximum potential life T is given

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[9] Incorporating the influence of effort choices on the magnitude of cash flow rate would be an interesting extension of this study. The case with both uncertain cash flows and uncertain failure rates is not tractable in the current setting.

[10] The more the effort expended by the agents, the higher the survival rate and the longer the expected lifespan of the project. Because the cash flow is constant at any instant, the longer the project survives, the greater the expected cash flows. In place of survival rate, we use a failure rate merely for modelling convenience.

[11] Indeed, all results hold if there is a countervailing term in the absolute failure rate (1) that characterizes inertia in the market, for example, $\alpha - 0.5\beta t^2$ with $\beta > 0$.

by[12]

$$F(T) = \frac{1}{2}(\lambda_1 + \lambda_2)T^2 - \lambda_1 \int_{s=0}^{T} \int_{\tau=0}^{s} e_1(\tau) d\tau ds - \lambda_2 \int_{s=0}^{T} \int_{\tau=0}^{s} e_2(\tau) d\tau ds. \tag{3}$$

A constant hazard rate is an alternative to describe the influence of shirking on the failure of the project. The hazard rate, which is conditional on survival until t, equals the absolute failure rate divided by the probability of surviving until t. Using hazard rates usually simplifies the stationarity analysis and makes the solution tractable in infinite horizon models. With a finite horizon, however, cross terms created by a constant hazard rate would complicate the algebra in this paper.

Assuming quadratic cost functions for all agents,[13] we write the expected social surplus as[14]

$$\Pi = b \int_{t=0}^{T} \big(1 - F(t)\big) dt - k_1 \int_{t=0}^{T} e_1(t)^2 dt - k_2 \int_{t=0}^{T} e_2(t)^2 dt - I.$$

The initial investment I is used for purchasing and installing equipment that is specifically geared to the restaurant at that location so that the resale value upon liquidation is essentially zero. At time t, the cash flow from the project is b while the probability that the project reaches t without failure is $1 - F(t)$. Thus, the expected cash flows are given by $b \int_{t=0}^{T} (1 - F(t)) dt$.[15] The total cost of exerting effort is $k_i \int_{t=0}^{T} e_i(t)^2 dt$ with a unit cost k_i for agent i, $i = 1, 2$. To focus on incentives, we assume that both the manager and the chef have linear utilities. As in most agency problems, the utility of consumption is separable from the disutility of effort.

Each agent's strategy satisfies an incentive compatibility (IC) constraint.[16] The manager chooses its continuous effort level (effort intensity), $0 \leq e_1(t) \leq 1$ for $t \in [0, T]$, to maximize

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[12] To ensure $F(t) \in [0, 1]$ for any effort level $\forall t \in [0, T]$, we assume $\frac{1}{2}(\lambda_1 + \lambda_2)T^2 \leq 1$. Assumption 1 summarizes technical conditions for the main result.

[13] The cost of effort needs to be strictly convex, differentiable, and increasing with $K(0) = 0$. We choose quadratic functions mainly because they simplify the algebra while rendering intuitive economic interpretations.

[14] We assume that the agents incur disutility of exerting effort over the entire feasible horizon of the project regardless of whether the project has failed. This is analogous to agents signing long-term labor contracts that are independent of the expected failure rate. This assumption changes the payoff little, but makes the impact of effort separable across time as it eliminates the cross-terms of effort that make an analytical solution impossible.

[15] The result is identical if future cash flows are discounted.

[16] We maximize the expected social surplus which is one way of finding an efficient (second-best) contract. This approach has the merit of focusing our effort on the agency problem. Alternatively, we could maximize the utility of one agent subject to the individual rationality constraint of the other agent to map out a frontier of efficient contracts. Or, we could think of agents as having endowments and entering into a bargaining problem.

the expected utility

$$\begin{aligned}
\Pi_1 &= \int_{t=0}^{T} c(t)\left(1-F(t)\right)dt - k_1\int_{t=0}^{T} e_1(t)^2dt - I_1\\
&= \int_{t=0}^{T} c(t)dt - \int_{t=0}^{T}\int_{\tau=t}^{T} c(\tau)d\tau\left((\lambda_1+\lambda_2)t - \lambda_1\int_{s=0}^{t} e_1(s)ds - \lambda_2\int_{s=0}^{t} e_2(s)ds\right)dt\\
&\quad - k_1\int_{t=0}^{T} e_1(t)^2dt - I_1\\
&= C_1 + \lambda_1\int_{t=0}^{T} e_1(t)\int_{s=t}^{T}\int_{\tau=s}^{T} c(\tau)d\tau ds dt - k_1\int_{t=0}^{T} e_1(t)^2dt,
\end{aligned}$$

where $C_1 \equiv -\frac{1}{2}(\lambda_1+\lambda_2)\int_{t=0}^{T} c(t)t^2dt+\lambda_2\int_{t=0}^{T}\int_{\tau=t}^{T} c(\tau)d\tau\int_{s=0}^{t} e_2(s)dsdt+\int_{t=0}^{T} c(t)dt-I_1$ collects terms independent of $e_1(t)$. The second equality is derived by substituting the failure rate (1) into Π_1 and changing the order of integration. The third equality is derived by rearranging terms and changing the order of integration. Note that the initial investment I_1 is sunk and does not affect effort choices.

Throughout the paper, we maximize the expected social surplus subject to the incentive compatibility constraint and the participation constraint of each agent. The first-order approach is adopted (see Holmström 1979): IC constraints are replaced with their first-order conditions. This is proper because the agents' maximization problems are convex: the linear utility of consumption is separable from the convex disutility of exerting effort.

By point-wise maximizing Π_1 with respect to $e_1(t)$, we have the optimal effort level of the manager:

$$e_1(t) = \frac{\lambda_1}{2k_1}\int_{s=t}^{T}\int_{\tau=s}^{T} c(\tau)d\tau ds. \tag{4}$$

Similarly, the optimal effort level of the chef $e_2(t)$ is

$$e_2(t) = \frac{\lambda_2}{2k_2}\int_{s=t}^{T}\int_{\tau=s}^{T}\big(b-c(\tau)\big)d\tau ds. \tag{5}$$

Notice that due to budget-balancing constraints, the more powerful the incentive given to one agent, the less powerful the incentive given to the other agent. Payments to the manager $c(\tau)\in[0,b]$, $\tau\in[0,T]$, are chosen to maximize the expected social surplus subject to incentive compatibility constraints (4) and (5). The initial investment I is sunk and does not affect the choice of $c(\tau)$. Thus, we ignore it.

We summarize the contract design problem in Problem 1.

Problem 1 *Choose payments to the manager* $0 \leq c(\tau) \leq b$ *for any* $t \in [0,T]$ *to maximize the expected social surplus*

$$\Pi = b\int_{t=0}^{T}(1-F(t))\,dt - k_1\int_{t=0}^{T}e_1(t)^2dt - k_2\int_{t=0}^{T}e_2(t)^2dt, \tag{6}$$

subject to IC constraints:

$$e_1(t) = \frac{\lambda_1}{2k_1}\int_{s=t}^{T}\int_{\tau=s}^{T}c(\tau)d\tau ds \quad \textit{and}\; e_2(t) = \frac{\lambda_2}{2k_2}\int_{s=t}^{T}\int_{\tau=s}^{T}\big(b-c(\tau)\big)d\tau ds,$$

where the cumulative failure rate at time t is

$$F(t) = \frac{1}{2}(\lambda_1+\lambda_2)t^2 - \lambda_1\int_{s=0}^{t}\int_{\tau=0}^{s}e_1(\tau)d\tau ds - \lambda_2\int_{s=0}^{t}\int_{\tau=0}^{s}e_2(\tau)d\tau ds.$$

Plugging IC constraints into the expected social surplus (6), changing the order of integration, and rearranging terms, we obtain

$$\Pi = C_2 - (s_1+s_2)\int_{t=0}^{T}\left(\int_{s=t}^{T}\int_{\tau=s}^{T}\left(c(\tau)-\frac{s_1}{s_1+s_2}b\right)d\tau ds\right)^2 dt, \tag{7}$$

where $C_2 \equiv bT - \frac{1}{6}(\lambda_1+\lambda_2)bT^3 + \frac{s_1^2+s_1s_2+s_2^2}{20(s_1+s_2)}b^2T^5$ collects terms independent of $c(\tau)$; $s_1 \equiv \frac{\lambda_1^2}{4k_1}$ and $s_2 \equiv \frac{\lambda_2^2}{4k_2}$ represent the severity of moral hazard of the manager and the chef, respectively. A greater s_i, i=1,2, corresponds to a greater influence multiplier λ_i or a smaller unit cost of exerting effort k_i. Therefore, the greater the s_i, the more severe the moral hazard problem faced by agent i.[17]

Given that the second terms of (7) are non-negative, the maximum of Π is reached when the second term equals zero. Namely, when $\int_{s=t}^{T}\int_{\tau=s}^{T}\big(c(\tau)-\frac{s_1}{s_1+s_2}b\big)d\tau ds = 0$, the objective

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[17] The severity of moral hazard measures the difference between the first best effort and the cost-minimizing effort. The first-best solution could be derived by maximizing the expected surplus subject to the participation constraint of each agent. Given that the agents have the right endowments and bargaining powers, the first best effort levels are $e_i(t)^{**} = \frac{\lambda_i}{4k_i}b(T-t)^2$.

function (7) is maximized point-wise. Therefore, the optimal cash allocation $c(\tau)$ for $\tau \in [0, T]$ is given by[18]

$$c(\tau)^* = \frac{s_1}{s_1 + s_2} b. \tag{8}$$

The optimal (second best) effort levels are[19]

$$e_1(t)^* = \frac{\lambda_1}{4k_1} \frac{s_1}{s_1 + s_2} b(T - t)^2 \quad \text{and} \quad e_2(t)^* = \frac{\lambda_2}{4k_2} \frac{s_2}{s_1 + s_2} b(T - t)^2. \tag{9}$$

We summarize technical conditions in Assumption 1 and the main result in Proposition 1.

ASSUMPTION 1 *The influence factors* λ_1 *and* λ_2*, unit costs of exerting effort* k_1 *and* k_2*, profit flow* b *and the potential lifespan* T *satisfy* (i) $\frac{1}{2}(\lambda_1 + \lambda_2)T^2 \leq 1$, (ii) $\frac{\lambda_1}{4k_1} \frac{s_1}{s_1+s_2} bT^2 \leq 1$, and (iii) $\frac{\lambda_2}{4k_2} \frac{s_2}{s_1+s_2} bT^2 \leq 1$.

Assumption 1(i) ensures by (3) that $F(t)$ (which is a probability) lies in $[0, 1]$ for all feasible effort levels for any $t \in [0, T]$. Assumption 1(ii) and (iii) ensure by (9) that $0 \leq e_1(t)^* \leq 1$ and $0 \leq e_2(t)^* \leq 1$ for any $t \in [0, T]$.

PROPOSITION 1 *Suppose Assumption 1 holds and that two agents exert continuous effort that affects the subsequent project failure. Then they share the proceeds of the project in fixed proportions of* $\frac{s_1}{s_1+s_2}$ *and* $\frac{s_2}{s_1+s_2}$*, where* $s_1 \equiv \frac{\lambda_1^2}{4k_1}$ *and* $s_2 \equiv \frac{\lambda_2^2}{4k_2}$ *denote the severity of the moral hazard problems of the two agents, respectively.*

Proof. See the derivation above.

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[18] In the case of three agents exerting ongoing effort that affects the subsequent project failure, the three agents divide the proceeds at any instant while the project is running in fixed proportions of

$$c_1(\tau) = \frac{s_1 s_2 + s_1 s_3 - s_2 s_3}{s_1 s_2 + s_1 s_3 + s_2 s_3}\, b, \quad c_2(\tau) = \frac{s_1 s_2 + s_2 s_3 - s_1 s_3}{s_1 s_2 + s_1 s_3 + s_2 s_3}\, b, \quad c_3(\tau) = \frac{s_1 s_3 + s_2 s_3 - s_1 s_2}{s_1 s_2 + s_1 s_3 + s_2 s_3}\, b,$$

where $c_i(\tau)$ is the proceeds allocated to agent i at time τ, $s_i \equiv \frac{\lambda_i^2}{4k_i}$ denotes the severity of the moral hazard problem faced by agent i for $i = 1, 2, 3$. The proof is similar to the proof of Proposition 1, provided upon request.

[19] The inefficiency in the second best solution is caused by the budget balancing constraint because each agent only gets a portion of the cash flows. A budget breaker (see Holmström 1982) may help achieve the first-best outcome. However, a budget breaker is infeasible if agents are wealth-constrained. Also, a budget breaker may introduce some other problems to the incentive scheme. For example, the budget breaker may collude with one agent to expropriate the other one.

This compensation contract resembles the equity-like claims where the shares are proportional to the relative severity of the moral hazard problems.[20] At any instant while the project is running, by (8), the manager receives a fraction of $\frac{s_1}{s_1+s_2}$ while the chef receives a fraction of $\frac{s_2}{s_1+s_2}$ of the profit flow.[21] The agent with more severe moral hazard (a larger $s_i \equiv \frac{\lambda_i^2}{4k_i}$) obtains a larger portion of the proceeds. Essentially, increasing k_i reduces the severity of the moral hazard problem of agent i. Reducing λ_i has the same effect on agent i's moral hazard as increasing k_i. As a result, the sharing rule above gives both agents proper incentives to exert effort.[22]

3 Deferred Compensation

In this section, we discuss a situation where the entrepreneur exerts effort at the outset when setting up the restaurant and the manager, who also works as a chef, expends ongoing effort while running the business.

The technical settings are as follows. The entrepreneur and the manager initially invest in the restaurant that generates a profit flow of b per unit of time while it's running. At any instant while the project is running, the manager receives $c(t)$ while the entrepreneur receives $b - c(t)$, where $0 \leq c(t) \leq b$ for any $t \in [0, T]$.[23] Given the incentive contract above, the

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[20] A fixed-fraction (fixed-proportion) contract could resolve many informational problems. Assuming stage financing with asymmetric information about the project profit, Admati and Pfleiderer (1994) show that a fixed-fraction contract induces the inside investor to make optimal investment decisions. In the framework of Myers and Majluf (1984), Dybvig and Zender (1991) compensate the manager with a fixed-fraction of firm's intrinsic value to mitigate the suboptimal investment problem due to informational asymmetry.

[21] The optimal contract works equally well in a more general setting where the proceeds of the project are time-dependent, such as: $c(t) = \frac{s_1}{s_1+s_2} b(t)$ for $t \in [0, T]$.

[22] An alternative proof of Proposition 1 using the Maximum Principle is available upon request. Essentially, the costate variable (Lagrangian multiplier) is identically zero under the fixed-fraction rule given in (8) and we have a Singular Control.

[23] Suppose alternatively that the proceeds of the project are deposited in an escrow account and are distributed to the agents upon project failure or at T if the project reaches its lifespan. As in the case discussed below in this section, the entrepreneur bears the cost of early project failure. The optimal incentive contract is characterized by the timing of project failure. If the project fails before a critical date t^*, the manager claims all the proceeds in the escrow account as a penalty for the entrepreneur's shirking because early failure is more likely due to the shirking of the entrepreneur at the outset. On the other hand, if the project fails after t^*, the entrepreneur claims all the proceeds. This penalizes the manager for not working hard because failure in later periods is more likely due to his shirking over time. Finally, if the project reaches its lifespan T, the two agents divide the proceeds in the escrow account. This serves as bonuses to incentivize both agents; particularly, to induce the manager to expend effort after t^*. The critical date t^* is determined by the relative severity of the moral hazard problems. This compensation schedule works as if inferences on agents' effort choices are made based on the

entrepreneur chooses an initial effort level of $e_0 \in [0, \phi]$ (ϕ denotes the ideal effort level) while the manager chooses ongoing effort $e_1(t)$ (a measurable function from $[0, T]$ to $[0, 1]$).

Assumption $c(t) \in [0, b]$ for any $t \in [0, T]$ represents financial constraints: neither agent injects cash into the project after the initial investment, and they cannot be paid by more than the profit available. If alternatively, the entrepreneur deposits a huge amount of cash up front, and if the project fails in a very initial stage, the manager would seize this deposit. As a result, the manager's incentives are only distorted in the very initial period. The first-best solution can be approximated arbitrarily closely. Dybvig and Lutz (1993) explicitly modelled this type of forcing contract in the context of product warranty. We are not going to discuss that here.

The absolute failure rate of the project at time s (probability density of the failure rate) is given by

$$f(s; e_0, e_1(\cdot); \phi, \lambda_1) = (\phi - e_0) + \lambda_1 \int_0^s \big(1 - e_1(\tau)\big) d\tau.$$

The entrepreneur's initial choice of effort e_0 has the same influence $(\phi - e_0)$ on the failure rate at all future times. The manager exerts effort $e_1(\tau)$ at time τ that has an impact $\lambda_1(1 - e_1(\tau))$ on the failure rate for all subsequent times.

The probability that the project fails before t is given by

$$\begin{aligned} F(t; e_0, e_1(\cdot); \phi, \lambda_1) &= \int_{s=0}^{t} f(s; e_0, e_1(\cdot); \phi, \lambda_1) ds \\ &= (\phi - e_0)t + \frac{1}{2}\lambda_1 t^2 - \lambda_1 \int_{s=0}^{t} \int_{\tau=0}^{s} e_1(\tau) d\tau ds. \end{aligned} \tag{10}$$

In later presentations, we use $f(s)$ as short for $f(s; e_0, e_1(\cdot); \phi, \lambda_1)$ and $F(t)$ for $F(t; e_0, e_1(\cdot); \phi, \lambda_1)$.

Using quadratic cost functions and omitting the initial investment, we write the expected social surplus of the project as

$$\Pi = b \int_{t=0}^{T} \big(1 - F(t)\big) dt - \gamma e_0^2 - k_1 \int_{t=0}^{T} e_1(t)^2 dt.$$

Term $b \int_{t=0}^{T} \big(1 - F(t)\big) dt$ denotes the expected payoff of the project. The unit cost of exerting effort is γ for the entrepreneur and is k_1 for the manager.

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time of project failure although agents' effort has been determined at the outset by the optimal compensation schedule (similar to the use of the likelihood function in compensating the agent in Holmström 1979).

In the following, we derive the effort levels as functions of the allocation rule through IC constraints. The entrepreneur chooses $e_0 \in [0, \phi]$ to maximize its expected surplus:

$$\begin{aligned}\Pi_0 =& \int_{t=0}^{T} \big(b - c(t)\big)\big(1 - F(t)\big)\, dt - \gamma e_0^2 \\ =& \int_{t=0}^{T} \big(b - c(t)\big) dt - \int_{t=0}^{T} \int_{\tau=t}^{T} (b - c(\tau))\, d\tau \big((\phi - e_0) + \lambda_1 t - \lambda_1 \int_{s=0}^{t} e_1(s) ds\big) dt - \gamma e_0^2 \\ =& C_3 + e_0 \int_{t=0}^{T} \int_{\tau=t}^{T} \big(b - c(\tau)\big) d\tau dt - \gamma e_0^2,\end{aligned}$$

where $\int_{t=0}^{T} \big(b - c(t)\big)\big(1 - F(t)\big) dt$ is the expected payments to the entrepreneur and $C_3 \equiv \int_{t=0}^{T} \big(b - c(t)\big) dt - \frac{1}{2} b\phi T^2 - \frac{1}{6}\lambda_1 b T^3$ collects terms independent of e_0.

As discussed in Section 2, the first-order approach applies. Thus, we replace IC constraints by their first-order conditions. Maximizing Π_0 with respect to e_0 yields

$$e_0 = \frac{1}{2\gamma} \int_{s=0}^{T} \int_{\tau=s}^{T} (b - c(\tau))\, d\tau ds. \tag{11}$$

Similarly, the expected surplus of the manager is given by

$$\begin{aligned}\Pi_1 &= \int_{t=0}^{T} c(t)\big(1 - F(t)\big) dt - k_1 \int_{t=0}^{T} e_1(t)^2 dt \\ &= C_4 + \lambda_1 \int_{t=0}^{T} e_1(t) \int_{s=t}^{T} \int_{\tau=s}^{T} c(\tau) d\tau ds dt - k_1 \int_{t=0}^{T} e_1(t)^2 dt,\end{aligned} \tag{12}$$

where $C_4 \equiv -\int_{t=0}^{T} \int_{\tau=t}^{T} c(\tau) d\tau (\phi - e_0 + \lambda_1 t) dt + \int_{t=0}^{T} c(t) dt$ collects terms independent of $e_1(t)$. Point-wise maximizing Π_1 with respect to $e_1(t)$ yields

$$e_1(t) = \frac{\lambda_1}{2k_1} \int_{s=t}^{T} \int_{\tau=s}^{T} c(\tau) d\tau ds. \tag{13}$$

We summarize the contract design problem in Problem 2.

Problem 2 *Choose payments to the manager $0 \leq c(\tau) \leq b$ for any $t \in [0, T]$ to maximize the expected social surplus*

$$\Pi = b \int_{t=0}^{T} \big(1 - F(t)\big) dt - \gamma e_0^2 - k_1 \int_{t=0}^{T} e_1(t)^2 dt, \tag{14}$$

subject to IC constraints:

$$e_0 = \frac{1}{2\gamma} \Big(\frac{1}{2} b T^2 - \int_{s=0}^{T} \int_{\tau=s}^{T} c(\tau) d\tau ds\Big)$$

and

$$e_1(t) = \frac{\lambda_1}{2k_1} \int_{s=t}^{T} \int_{\tau=s}^{T} c(\tau) d\tau ds,$$

where the cumulative failure rate at time t is

$$F(t) = (\phi - e_0)t + \frac{1}{2}\lambda_1 t^2 - \lambda_1 \int_{s=0}^{t} \int_{\tau=0}^{s} e_1(\tau) d\tau ds.$$

Changing the order of integration, we have

$$\int_{s=t}^{T} \int_{\tau=s}^{T} c(\tau) d\tau ds = \int_{s=t}^{T} c(s)(s-t) ds. \tag{15}$$

Hence, by (11) and (13), deferring compensation to the manager could increase $\int_{s=t}^{T} c(s) s ds$ for some set t of positive measure while keeping $\int_{s=0}^{T} c(s) s ds$ unchanged. This gives the manager better incentives while not damaging the incentives of the entrepreneur. As a result, the expected social surplus will be greater under the following policy that defers payments to the manager:

$$c(t) = \begin{cases} 0, & \text{if } t < t_d, \\ b, & \text{otherwise;} \end{cases} \tag{16}$$

where $t_d \in (0, T)$ is a critical date before which the entrepreneur receives all the cash flows and after which the manager claims all the proceeds. This incentive contract is indeed optimal. It is formally proved using the Maximum Principle. An alternative proof by Dominance Claims is available upon request.

Technical conditions are summarized in Assumption 2 and the optimal compensation contract is given in Proposition 2.

ASSUMPTION 2 *Parameters satisfy* (i) $\phi T + \frac{1}{2}\lambda_1 T^2 \leq 1$, (ii) $\frac{bT^2}{4\gamma} \leq \phi$, and (iii) $\frac{\lambda_1}{4k_1} b T^2 \leq 1$.

Assumption 2(i) ensures by (10) that $F(t)$ (which is a probability) lies in $[0, 1]$ for all $t \in [0, T]$ and all feasible effort levels. Assumption 2(ii) ensures by (11) that $e_0 \leq \phi$ and (iii) ensures by (13) that $e_1(t) \leq 1$ for any $t \in [0, T]$.

PROPOSITION 2 *Suppose Assumption 2 holds and that the entrepreneur expends initial effort while the manager exerts effort over time. Then there exists a critical date* $t_d \in (0, T)$ *given by*

$$t_d^3 + \frac{1}{\gamma s_1} t_d^2 - \frac{1}{\gamma s_1} T^2 = 0, \tag{17}$$

such that (i) *the entrepreneur receives all the proceeds prior to* t_d*; namely,* $c(t) = 0$ *for* $t < t_d$*; and* (ii) *the manager receives all the proceeds after* t_d*; namely,* $c(t) = b$ *for* $t \geq t_d$*.*

Intuitively, the up-front effort of the entrepreneur is sunk once the project begins. Therefore, the incentive to exert effort is not affected by the timing of payments as long as the present value of compensation is the same. In contrast, because the manager would shirk once his payments stop, we need to defer his compensation to keep him working hard throughout the life of the restaurant. In the optimal contract, all the proceeds of the project go to the entrepreneur before a critical date and the manager bears all the risks in the firm value afterwards. As a result, before this critical date, the manager has to put in effort to keep the project running so that he can get payments later on. After this critical date, the manager works hard because he is the sole claimant of the cash flows.

Without further institutional details, this optimal compensation contract can be interpreted as employee stock options. The entrepreneur owns the restaurant initially while the manager has a call option on all the profit flows of the restaurant. The exercise price of the option is the cumulative cash flows prior to the critical date. Exercising the option is cashless, that is, the manager receives the value of the option: all profit flows after the critical date.

Comparative statics is derived from the first order condition of (17). We find that t_d increases in T so that the longer the horizon T, the more the cash flows (in absolute value) are allocated to the entrepreneur (a larger t_d). In addition, t_d decreases in the ratio $\gamma s_1 \equiv \gamma \frac{\lambda_1^2}{4k_1}$. Recall that a larger γ indicates less severe moral hazard for the entrepreneur and a larger s_1 indicates more severe moral hazard for the manager. Therefore, if we increase γs_1 which indicates that the moral hazard problem of the manager is getting more severe compared to that of the entrepreneur (a larger γs_1), then more proceeds will be allocated to the manager (a smaller t_d) to improve his incentives.

Proof of Proposition 2.

For notational convenience, define

$$x_1(t) \equiv \int_{s=t}^{T} \int_{\tau=s}^{T} c(\tau) d\tau ds.$$

Substituting IC constraints (11) and (13) into the objective function (14) and omitting the terms independent of $x_1(t)$, we have

$$\bar{\Pi} = -\frac{1}{4\gamma} x_1(0)^2 + s_1 b \int_{t=0}^{T} x_1(t)(T-t)^2 dt - s_1 \int_{t=0}^{T} x_1(t)^2 dt.$$

We need to choose $c(t)$ (a measurable function from $[0, T]$ to $[0, b]$) to maximize $\bar{\Pi}$. In the following, we will apply the Maximum Principle to solve this problem. Define $x_2(t) = \dot{x}_1(t)$, then $\dot{x}_2(t) = c(t)$. The optimization problem is transformed to

Problem 2A. *Choose* $0 \leq c(t) \leq b$ *for any* $t \in [0, T]$ *to maximize* $\bar{\Pi}$, *subject to system dynamics:*

$$\dot{x}_1(t) = x_2(t), \qquad \dot{x}_2(t) = c(t),$$

and terminal conditions:

$$x_1(T) = 0, \quad x_2(T) = 0.$$

The Hamiltonian is

$$H = s_1 b x_1(t)(T-t)^2 - s_1 x_1(t)^2 + \lambda_1 x_2(t) + \lambda_2 c(t),$$

and the initial value is $\phi(x_1(0)) = \frac{1}{4\gamma} x_1(0)^2$.

The Hamiltonian is linear in the control variable $c(t)$. We should expect a Bang-Bang Control – at any instant, a single agent claims all the proceeds – or a Singular Control. We will prove that the former is optimal.

Costate variables (Lagrangian multipliers) satisfy:

$$\begin{aligned}
\dot{\lambda}_1(t) &= -\frac{\partial H}{\partial x_1(t)} = -s_1 b(T-t)^2 + 2s_1 x_1(t), \\
\lambda_1(0) &= \frac{\partial \phi(x_1(0))}{\partial x_1(0)} = \frac{1}{2\gamma} x_1(0), \\
\dot{\lambda}_2(t) &= -\frac{\partial H}{\partial x_2(t)} = -\lambda_1(t), \\
\lambda_2(0) &= 0.
\end{aligned} \tag{18}$$

We prove the optimality in three steps. First, we show that $\dot{\lambda}_2(0) < 0$. Second, we show that $\ddot{\lambda}_2(t) > 0$ for any t at which $\lambda_2(t) < 0$. Essentially, $\lambda_2(t)$ is convex, it starts at zero and becomes negative first, then crosses zero from below. Third, we solve for the crossing point $t_d \in (0, T)$: $\lambda_2(t_d) = 0$.

The costate variable and the payments to the manager are depicted in Figure 1:

$$\lambda_2(t) \begin{cases} < 0 & \text{if } t < t_d, \\ = 0 & \text{if } t = t_d, \\ > 0 & \text{otherwise.} \end{cases} \tag{19}$$

The optimal allocation rule is[24]

$$c(t) = \begin{cases} 0 & \text{if } t < t_d, \\ b & \text{otherwise.} \end{cases} \tag{20}$$

LEMMA 1 *$\dot{\lambda}_2(0) < 0$.*

Proof. $\dot{\lambda}_2(0) = -\lambda_1(0) = -\frac{1}{2\gamma} x_1(0) \equiv -\frac{1}{2\gamma} \int_{s=0}^{T} \int_{\tau=s}^{T} c(\tau) d\tau ds \le 0$ because $c(\tau) \ge 0$ for any $\tau \in [0, T]$. We will prove by contradiction that $\dot{\lambda}_2(0) < 0$. Suppose $\dot{\lambda}_2(0) = 0$. Then, we have $c(\tau) = 0$ for any $\tau \in (0, T]$. This implies by (18) that $\dot{\lambda}_1(\tau) < 0$ and thus $\ddot{\lambda}_2(\tau) = -\dot{\lambda}_1(\tau) > 0$. Given that $\lambda_2(t)$ is convex, $\lambda_2(0) = 0$, and $\dot{\lambda}_2(0) = 0$, we should have $\lambda_2(\tau) > 0$ for any $\tau \in (0, T]$, which in turn requires that $c(\tau) = b$ for any $\tau \in (0, T]$, a contradiction.

LEMMA 2 *$\ddot{\lambda}_2(t) > 0$ for any t at which $\lambda_2(t) < 0$.*

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[24] *Claim.* If $\lambda_2(t) = 0$, then $c(t) = b$.

Proof. If $\lambda_2(t) = 0$, then all the derivatives of $\lambda_2(t)$ equal zero. In particular, $\dddot{\lambda}_2(t) = 0$ yields $c(t) = b$. This concludes the proof.

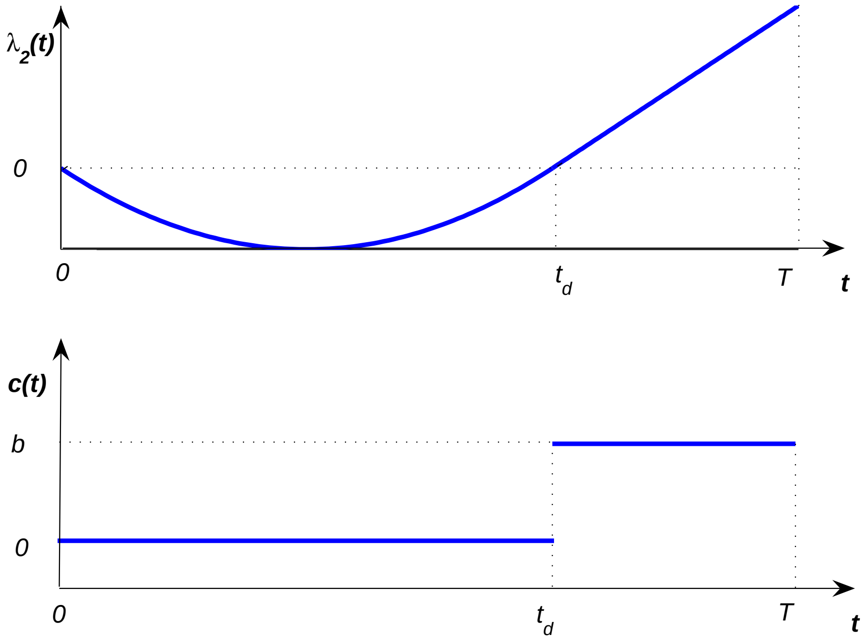

Figure 1: This figure depicts the switching function (Lagrangian multiplier) $\lambda_2(t)$ and the control $c(t) \in [0, b]$ (payments to the manager) over the project lifespan. $\lambda_2(t)$ is convex. It starts at zero, goes negative first and crosses zero once from below at a critical date $t_d \in (0, T)$. Correspondingly, $c(t)$ is zero (the minimum payment) prior to t_d and is b (the maximum payment) after t_d.

Proof. $\ddot{\lambda}_2(t) = -\dot{\lambda}_1(t) = s_1 b(T-t)^2 - 2s_1 x_1(t) = s_1 b(T-t)^2 - 2s_1 \int_{s=t}^{T} \int_{\tau=s}^{T} c(\tau) d\tau ds \geq 0$ because $c(\tau) \leq b$ for any $\tau \in [0, T]$. We will prove by contradiction that $\ddot{\lambda}_2(t) > 0$ for any $\lambda_2(t) < 0$. Suppose $\ddot{\lambda}_2(t) = 0$ for some t at which $\lambda_2(t) < 0$, then by (18) we have $c(\tau) = b$ for any $\tau \in (t, T]$. However, given that $\lambda_2(t) < 0$, there exists $\delta > 0$ such that $\lambda_2(\tau) < 0$ for any $\tau \in [t, t+\delta]$. This requires $c(\tau) = 0$ for $\tau \in [t, t+\delta]$, a contradiction.

Because $\lambda_2(0) = 0$, $\dot{\lambda}_2(0) < 0$, and $\ddot{\lambda}_2(t) > 0$ if $\lambda_2(t) < 0$, $\lambda_2(t)$ will cross zero from below. Denote the crossing point t_d. In the following, we determine t_d by $\lambda_2(t_d) = 0$, show that t_d falls in $(0, T)$ and it is unique.

By (20), the state variable satisfies

$$\begin{aligned} x_1(t) &= \begin{cases} \frac{1}{2}b\big((T-t)^2 - (t_d - t)^2\big) & \text{if } t < t_d, \\ \frac{1}{2}b(T-t)^2 & \text{otherwise;} \end{cases} \\ x_1(0) &= \frac{1}{2}b(T^2 - t_d^2). \end{aligned} \tag{21}$$

Plugging (21) into (18), we have the costate variable satisfying

$$\begin{aligned} \dot{\lambda}_1(t) &= \begin{cases} -s_1 b(t_d - t)^2, & \text{if } t < t_d, \\ 0, & \text{otherwise;} \end{cases} \\ \lambda_1(0) &= \frac{1}{4\gamma}b(T^2 - t_d^2). \end{aligned} \tag{22}$$

Using (18) and (22), we rewrite the switching function $\lambda_2(t)$ for $t \leq t_d$ as

$$\begin{aligned} \lambda_2(t) &= \lambda_2(0) + \int_{s=0}^{t} \dot{\lambda}_2(s)ds \\ &= \lambda_2(0) - \int_{s=0}^{t} \lambda_1(s)ds \\ &= \lambda_2(0) - \int_{s=0}^{t} \left(\lambda_1(0) + \int_{\tau=0}^{s} \dot{\lambda}_1(\tau)d\tau\right) ds \\ &= -\frac{1}{4\gamma}b(T^2 - t_d^2)t + \frac{1}{12}s_1 b\big(6t_d^2t^2 - 4t_dt^3 + t^4\big). \end{aligned} \tag{23}$$

The condition that $\lambda_2(t_d) = 0$ yields $t_d = 0$ or

$$P(t_d) \equiv t_d^3 + \frac{1}{\gamma s_1}t_d^2 - \frac{1}{\gamma s_1}T^2 = 0. \tag{24}$$

LEMMA 3 *There is a unique solution* $t_d^* \in (0, T)$ *to* (24).

Proof. Because $P(t_d|t_d = 0) = -\frac{1}{\gamma s_1}T^2 < 0$ and $P(t_d|t_d = T) = T^3 > 0$, there exists at least one solution in $(0, T)$ to (24). Additionally, we have $P'(t_d) = 3t_d^2 + \frac{2}{\gamma s_1}t_d > 0$ for any $t_d > 0$. Hence, this solution is unique.

For any $t \in (t_d, T]$, $\ddot{\lambda}_2(t) = 0$, $\dot{\lambda}_2(t) > 0$, and $\lambda_2(t) > 0$, thus $c(t) = b$ is the control. Therefore, the proposed control (20) satisfies the necessary condition of the Maximum Principle.

Q.E.D.

4 Three-Sided Moral Hazard

This section provides a full model with three-sided moral hazard. The entrepreneur exerts effort e_0 at the outset when setting up the restaurant while the manager and the chef put in effort $e_1(t)$ and $e_2(t)$ over time to achieve cash flows.

Notice that the three-sided moral hazard model is not a trivial extension of the two-sided moral hazard models in Section 2 and Section 3. In general, we don't know the properties of the optimal contract except that the entrepreneur always claims all proceeds early on. In a special case when the manager and the chef face equally severe moral hazard problems, the entrepreneur claims all proceeds prior to a critical date; while the manager and the chef split cash flows equally afterwards. In this case, the shares of the manager and the chef are indeed proportional to the relative severity of the moral hazard problems.[25]

The absolute failure rate at time s (the probability density of failure) is given by

$$f(s) = (\phi - e_0) + \lambda_1 \int_{\tau=0}^{s} \big(1 - e_1(\tau)\big) d\tau + \lambda_2 \int_{\tau=0}^{s} \big(1 - e_2(\tau)\big) d\tau,$$

where subscripts "0", "1", and "2" represent the entrepreneur, the manager, and the chef, respectively. The entrepreneur's effort choice e_0 has the same influence $(\phi - e_0)$ on the failure rate at all future times. The manager and the chef put in effort $e_1(\tau)$ and $e_2(\tau)$ that have impacts $\lambda_1(1 - e_1(\tau))$ and $\lambda_2(1 - e_2(\tau))$, respectively, on the failure rate for all subsequent times.

The probability of failure before time t is given by

$$F(t) = (\phi - e_0)t + \frac{1}{2}(\lambda_1 + \lambda_2)t^2 - \lambda_1 \int_{s=0}^{t} \int_{\tau=0}^{s} e_1(\tau) d\tau ds - \lambda_2 \int_{s=0}^{t} \int_{\tau=0}^{s} e_2(\tau) d\tau ds. \quad (25)$$

Suppose the compensation for the manager and the chef are $c_1(t)$ and $c_2(t)$, respectively, for any $t \in [0, T]$. The residual $b - c_1(t) - c_2(t)$ goes to the entrepreneur. The limited-liability constraint applies. Using quadratic cost functions and omitting initial investments, we have the following maximization problem:

Problem 3 *Choose payments to the manager and the chef* $c_1(\tau)$ *and* $c_2(\tau)$ (measurable functions from $[0, T]$ to $[0, b]$, satisfying $c_1(\tau) + c_2(\tau) \leq b$) *to maximize the expected social surplus:*

$$\Pi = b \int_{t=0}^{T} \big(1 - F(t)\big) dt - \gamma e_0^2 - k_1 \int_{t=0}^{T} e_1(t)^2 dt - k_2 \int_{t=0}^{T} e_2(t)^2 dt, \quad (26)$$

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[25] We can also prove that if the manager's moral hazard differs significantly from the chef's, then there exists a second critical date after the first one. The agent with less severe moral hazard claims all the cash flows between the two dates. This contract keeps the agent with more severe moral hazard working hard throughout the life time o the restaurant.

subject to IC constraints:

$$
\begin{aligned}
e_0 &= \frac{1}{2\gamma}\int_{s=0}^{T}\int_{\tau=s}^{T}\Big(b - c_1(\tau) - c_2(\tau)\Big)d\tau ds,\\
e_1(t) &= \frac{\lambda_1}{2k_1}\int_{s=t}^{T}\int_{\tau=s}^{T} c_1(\tau)d\tau ds \qquad (27)\\
e_2(t) &= \frac{\lambda_2}{2k_2}\int_{s=t}^{T}\int_{\tau=s}^{T} c_2(\tau)d\tau ds;
\end{aligned}
$$

where the cumulative failure rate at time t is

$$F(t) = (\phi - e_0)t + \frac{1}{2}(\lambda_1 + \lambda_2)t^2 - \int_{s=0}^{t}\int_{\tau=0}^{s}\big(\lambda_1 e_1(\tau) + \lambda_2 e_2(\tau)\big)d\tau ds.$$

The IC constraints are derived, as in Sections 2 and 3, using the first-order conditions of the three agents' maximization problems. Essentially, all three agents are protected by limited liability and they divide all the cash flows of the project when they become available.[26] Effort levels are derived using the first-order approach as in Sections 2 and 3. Observe from (27) that the entrepreneur's incentives are independent of the timing of payments while the manager and the chef work harder their payments are deferred. In the optimal contract, the entrepreneur should receive all the proceeds early on while the manager and the chef split the proceeds afterwards:

$$c_1(t) + c_2(t) = \begin{cases} 0, & \text{if } t < t_d,\\ b, & \text{otherwise.}\end{cases}$$

Indeed, if the manager and the chef have the same magnitudes of moral hazard, they split the profit flow equally after t_d. We summarize technical conditions in Assumption 3 and formally state the result in Proposition 3. The proof applying the Maximum Principle is in the Appendix.

ASSUMPTION 3 *Parameters satisfy* (i) $\phi T + \frac{1}{2}(\lambda_1 + \lambda_2)T^2 \leq 1$, (ii) $\frac{bT^2}{4\gamma} \leq \phi$, (iii) $\frac{\lambda_1}{4k_1}bT^2 \leq 1$, and (iv) $\frac{\lambda_2}{4k_2}bT^2 \leq 1$.

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[26] Alternatively, the entrepreneur may exert effort not only at the outset but also over time. In the restaurant example, the so-called *entrepreneur* sets up the restaurant and manages its daily operations. Meanwhile, a chef works over time to guarantee the quality of the food served in the restaurant. In the optimal contract, the payments to the entrepreneur differs from the sum of the payments to an agent with pure up-front effort and the payments to an agent with pure continuous effort. In general, the model remains unsolved. In a special case when the entrepreneur's up-front effort is relatively important, the entrepreneur collects all cash flows before a critical date while the chef receives all proceeds afterwards. This critical date is determined by the relative severity of the moral hazard problems.

Assumption 3(i) ensures by (25) that $F(t)$ (which is a probability) lies in $[0,1]$ for all feasible effort levels for any $t \in [0,T]$. Assumptions 3(ii), (iii), and (iv) ensure by (27) that $e_0 \le \phi$, $e_1(t) \le 1$, and $e_2(t) \le 1$ for any $t \in [0,T]$.

PROPOSITION 3 *Suppose Assumption 3 holds and that the entrepreneur expends initial effort while the manager and the chef exert effort over time. Then there exists a unique critical date* $t_d \in (0,T)$ *given by*

$$2t_d^3 - 2Tt_d^2 + 3T^2 t_d - \frac{3}{\gamma s_1}(T^2 - t_d^2) = 0,$$

such that (i) *the entrepreneur claims all the proceeds prior to* t_d*; namely,* $c_1(t) = c_2(t) = 0$ *for* $t < t_d$*;* (ii) *the manager and the chef split the proceeds after* t_d*; namely,* $c_1(t) + c_2(t) = b$ *for* $t \ge t_d$*; and* (iii) *if the manager and the chef face equally severe moral hazard, i.e.,* $s_1 \equiv \frac{\lambda_1^2}{4k_1} = s_2 \equiv \frac{\lambda_2^2}{4k_2}$*, then they split the proceeds equally; namely,* $c_1(t) = c_2(t) = \frac{1}{2}b$ *for* $t \ge t_d$.

Proof. See the Appendix.

Comparative statics is derived from the first order condition (40). We find that t_d decreases in the ratio $\gamma s_1 = \gamma \frac{\lambda_1^2}{4k_1}$. The greater this ratio, the more severe the moral hazard problem of the manager and the chef is. Therefore, more cash flows are allocated to them (a smaller t_d). In addition, we have $\frac{\partial t_d}{\partial T} > 0$ if $\frac{1}{\gamma s_1} > \frac{2}{3}T$. This indicates that if the moral hazard of the entrepreneur is relatively severe compared to that of the manager and the chef, the longer the project's maximum potential life, the more cash flow rights are allocated to the entrepreneur (a larger t_d).

5 First-Best Comparison

5.1 Fixed Proportion

The first best efforts are

$$\begin{aligned} e_1^{fb}(t) &= \frac{\lambda_1}{4k_1} b(T-t)^2 \\ e_2^{fb}(t) &= \frac{\lambda_2}{4k_2} b(T-t)^2. \end{aligned} \tag{28}$$

The first best social surplus is

$$\Pi^{fb} = bT - \frac{1}{6}\left(\lambda_1 + \lambda_2\right) bT^3 + \frac{1}{20}(s_1 + s_2)b^2T^5.$$

The second best efforts are

$$\begin{aligned} e_1^{sb}(t) &= \frac{\lambda_1}{4k_1}\frac{s_1}{s_1+s_2}b(T-t)^2 \\ e_2^{sb}(t) &= \frac{\lambda_2}{4k_2}\frac{s_2}{s_1+s_2}b(T-t)^2. \end{aligned} \tag{29}$$

The first best social surplus is

$$\Pi^{sb} = bT - \frac{1}{6}\left(\lambda_1 + \lambda_2\right) bT^3 + \frac{1}{20}\frac{s_1^2 + s_1 s_2 + s_2^2}{s_1 + s_2}b^2T^5.$$

The welfare loss $(\Pi^{fb} - \Pi^{sb})$ is

$$\frac{1}{20}\frac{s_1 s_2}{s_1 + s_2}b^2T^5.$$

5.2 Deferred Compensation

The first best efforts are

$$\begin{aligned} e_0^{fb} &= \frac{1}{4\gamma}bT^2 \\ e_1^{fb}(t) &= \frac{\lambda_1}{4k_1}b(T-t)^2. \end{aligned} \tag{30}$$

The first best social surplus is

$$\Pi^{fb} = bT - \frac{1}{2}\phi bT^2 - \frac{1}{6}\lambda_1 bT^3 + \frac{1}{16\gamma}b^2T^4 + \frac{1}{20}s_1 b^2T^5.$$

The second best efforts are

$$\begin{aligned} e_0^{sb} &= \frac{1}{4\gamma}bt_d^2 \\ e_1^{sb}(t) &= \begin{cases} \frac{\lambda_1}{4k_1}b\left((T-t)^2 - (t_d - t)^2\right), & \text{if } t < t_d, \\ \frac{\lambda_1}{4k_1}b(T-t)^2, & \text{otherwise;} \end{cases} \end{aligned} \tag{31}$$

The first best social surplus is

$$\Pi^{sb} = bT - \frac{1}{2}\phi bT^2 - \frac{1}{6}\lambda_1 bT^3 + \frac{1}{8\gamma}b^2t_d^2T^2 - \frac{1}{16\gamma}b^2t_d^4 + \frac{1}{20}s_1 b^2(T^5 - t_d^5).$$

The welfare loss $(\Pi^{fb} - \Pi^{sb})$ is

$$\frac{1}{16\gamma}b^2(T^2 - t_d^2)^2 + \frac{1}{20}s_1 b^2 t_d^5.$$

5.3 Three-sided Moral Hazard

The first best efforts are

$$\begin{aligned} e_0^{fb} &= \frac{1}{4\gamma} bT^2 \\ e_1^{fb}(t) &= \frac{\lambda_1}{4k_1} b(T-t)^2 \\ e_2^{fb}(t) &= \frac{\lambda_2}{4k_2} b(T-t)^2. \end{aligned} \tag{32}$$

The first best social surplus is

$$\Pi^{fb} = bT - \frac{1}{2}\phi bT^2 - \frac{1}{6}(\lambda_1+\lambda_2) bT^3 + \frac{1}{16\gamma} b^2T^4 + \frac{1}{20}(s_1+s_2) b^2T^5.$$

The second best efforts are

$$\begin{aligned} e_0^{sb} &= \frac{1}{4\gamma} bt_d^2 \\ e_1^{sb}(t) &= \begin{cases} \frac{\lambda_1}{8k_1} b\left((T-t)^2 - (t_d-t)^2\right), & \text{if } t < t_d, \\ \frac{\lambda_1}{8k_1} b(T-t)^2, & \text{otherwise;} \end{cases} \\ e_2^{sb}(t) &= \begin{cases} \frac{\lambda_2}{8k_2} b\left((T-t)^2 - (t_d-t)^2\right), & \text{if } t < t_d, \\ \frac{\lambda_2}{8k_2} b(T-t)^2, & \text{otherwise.} \end{cases} \end{aligned} \tag{33}$$

The first best social surplus is

$$\Pi^{sb} = bT - \frac{1}{2}\phi bT^2 - \frac{1}{6}(\lambda_1+\lambda_2) bT^3 + \frac{1}{8\gamma} b^2t_d^2T^2 - \frac{1}{16\gamma} b^2t_d^4 + \frac{1}{40}(s_1+s_2) b^2(T^5 - t_d^5).$$

The welfare loss ($\Pi^{fb} - \Pi^{sb}$) is

$$\frac{1}{16\gamma} b^2(T^2 - t_d^2)^2 + \frac{1}{40}(s_1+s_2) b^2(T^5 + t_d^5).$$

6 Conclusion

We have proposed a dynamic incentive contract to solve a three-sided moral hazard problem where agents expend effort at different times. In the optimal contract, the timing of compensation reflects the timing of effort. In particular, the agent expending up-front effort exclusively claims proceeds early on and the agents exerting effort over time exclusively claim proceeds in later periods. Additionally, the payments to an agent increase with the relative severity of his moral hazard problem.

The model developed in this paper is suggestive of an approach to understanding a broader set of contracting problems in financial markets. This approach should be particularly useful when the timing of effort impacts the form of the optimal contract. Possible applications include deferred compensation, Management Buyouts (MBOs), and employee stock options.

7 Appendix

Proof of Proposition 3.

Substituting IC constraints (27) into (25) and (26) and omitting the terms independent of x_1 and y_1, we obtain the following expected social surplus:

$$\begin{aligned}\hat{\Pi} = -\frac{1}{4\gamma}(x_1(0)+y_1(0))^2 + &s_1 b\int_{t=0}^{T} x_1(t)(T-t)^2dt - s_1\int_{t=0}^{T} x_1(t)^2dt\\ &+s_2 b\int_{t=0}^{T} y_1(t)(T-t)^2dt - s_2\int_{t=0}^{T} y_1(t)^2dt,\end{aligned}$$

where

$$x_1(t) \equiv \int_{s=t}^{T}\int_{\tau=s}^{T} c_1(\tau)d\tau ds, \qquad y_1(t) \equiv \int_{s=t}^{T}\int_{\tau=s}^{T} c_2(\tau)d\tau ds.$$

Define $x_2(t) = \dot{x}_1(t)$ and $y_2(t) = \dot{y}_1(t)$, then we have $\dot{x}_2(t) = c_1(t)$ and $\dot{y}_2(t) = c_2(t)$. The optimization problem is transformed to

Problem 3A. *Choose* $(c_1(t), c_2(t))$ *for any* $t \in [0, T]$ *from the admissible set of control:*

$$c_1(t) \geq 0, \quad c_2(t) \geq 0, \qquad c_1(t) + c_2(t) \leq b$$

to maximize $\hat{\Pi}$, *subject to system dynamics:*

$$\begin{aligned}\dot{x}_1(t) &= x_2(t), &\qquad \dot{x}_2(t) &= c_1(t),\\ \dot{y}_1(t) &= y_2(t), &\qquad \dot{y}_2(t) &= c_2(t),\end{aligned}$$

and terminal conditions:

$$\begin{aligned}x_1(T) &= 0, \quad & x_2(T) &= 0,\\ y_1(T) &= 0, \quad & y_2(T) &= 0.\end{aligned}$$

We apply the Maximum Principle to solve this problem. The Hamiltonian is

$$\begin{aligned}H =& s_1 b x_1(t)(T-t)^2 - s_1 x_1(t)^2 + s_2 b y_1(t)(T-t)^2 - s_2 y_1(t)^2\\ &+ \lambda_1 x_2(t) + \lambda_2 c_1(t) + \mu_1 y_2(t) + \mu_2 c_2(t),\end{aligned}$$

and the initial value is $\phi(x_1(0), y_1(0)) = \frac{1}{4\gamma}\big(x_1(0) + y_1(0)\big)^2$.

Costate variables satisfy

$$\begin{aligned}
\dot{\lambda}_1(t) &= -\frac{\partial H}{\partial x_1(t)} = -s_1 b(T-t)^2 + 2s_1 x_1(t),\\
\lambda_1(0) &= \frac{\partial \phi(x_1(0), y_1(0))}{\partial x_1(0)} = \frac{1}{2\gamma}\big(x_1(0) + y_1(0)\big),\\
\dot{\lambda}_2(t) &= -\frac{\partial H}{\partial x_2(t)} = -\lambda_1(t),\\
\lambda_2(0) &= 0;
\end{aligned} \tag{34}$$

and

$$\begin{aligned}
\dot{\mu}_1(t) &= -\frac{\partial H}{\partial y_1(t)} = -s_2 b(T-t)^2 + 2s_2 y_1(t),\\
\mu_1(0) &= \frac{\partial \phi(x_1(0), y_1(0))}{\partial y_1(0)} = \frac{1}{2\gamma}\big(x_1(0) + y_1(0)\big),\\
\dot{\mu}_2(t) &= -\frac{\partial H}{\partial y_2(t)} = -\mu_1(t),\\
\mu_2(0) &= 0.
\end{aligned} \tag{35}$$

By (34) and (35), we know that in the knife-edge case when $s_1 = s_2$, $\lambda_1(t) = \mu_1(t)$ and $\lambda_2(t) = \mu_2(t)$ for any $t \in [0, T]$. We will focus on conditions for $\lambda_2(t)$ in the rest of this section.

Notice that the Hamiltonian is linear in control variables $c_1(t)$ and $c_2(t)$. The building blocks of the optimal control can only be a Bang-Bang Control (at any instant, a single agent claims all the proceeds) or a Singular Control (the manager and the chef split the cash flows in proportions of $\frac{s_1}{s_1+s_2}b$ and $\frac{s_2}{s_1+s_2}b$.[27])

In the following, we will show (1) $\dot{\lambda}_2(0) < 0$ and (2) $\ddot{\lambda}_2(t) > 0$ for $t \in [0, T]$. Therefore, $\lambda_2(t)$ starts at zero and becomes negative first. We then provide the condition that determines the moment t_d at which $\lambda_2(t)$ crosses zero from below.

The optimal allocation rule is

$$\begin{cases} c_1(t) = c_2(t) = 0, & \text{if } \lambda_2(t) < 0 (t < t_d),\\ c_1(t) = c_2(t) = \frac{1}{2}b, & \text{if } \lambda_2(t) > 0 (t \geq t_d). \end{cases} \tag{36}$$

Essentially, the entrepreneur claims all the cash flows prior to a critical date t_d; afterwards, the manager and the chef split the proceeds equally if $s_1 = s_2$.

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[27] *Claim.* If $\lambda_2(t) = \mu_2(t) > 0$, we have $c_1(t) = \frac{s_1}{s_1+s_2}b$ and $c_2(t) = \frac{s_2}{s_1+s_2}b$.

Proof. If $\lambda_2(t) = \mu_2(t) > 0$, we have $c_1(t) + c_2(t) = b$. Define $\psi(t) \equiv \lambda_2(t) - \mu_2(t) = 0$, then all the derivatives of $\psi(t)$ equal zero. In particular, $\dddot{\psi}(t) = 0$ yields $c_1(t) = \frac{s_1}{s_1+s_2}b$. This concludes the proof.

LEMMA 4 $\dot{\lambda}_2(0) = \dot{\mu}_2(0) < 0$.

Proof. $\dot{\lambda}_2(0) = -\lambda_1(0) = -\frac{1}{2\gamma}\left(x_1(0) + y_1(0)\right) \leq 0$ by definitions of $x_1(0)$ and $y_1(0)$. We will prove by contradiction that $\dot{\lambda}_2(0) < 0$. Suppose not, then $x_1(0) + y_1(0) = 0$ implies that $c_1(t) = c_2(t) = 0$ for any $t \in (0, T]$. Thus, $x_1(t) = y_1(t) \equiv 0$, which implies that $\ddot{\lambda}_2(t) = -\dot{\lambda}_1(t) = s_1 b(T-t)^2 > 0$. Given that $\lambda_2(0) = 0$ and $\dot{\lambda}_2(0) = 0$, we have $\lambda_2(t) > 0$ for any $t \in (0, T]$. Recall that $\lambda_2(t) \equiv \mu_2(t)$, we have $c_1(t) = c_2(t) = \frac{1}{2}b$ for any $t \in (0, T]$, a contradiction.

LEMMA 5 $\ddot{\lambda}_2(t) = \ddot{\mu}_2(t) > 0$ *for any* $t \in [0, T]$.

Proof. $\ddot{\lambda}_2(t) = -\dot{\lambda}_1(t) = s_1 b(T-t)^2 - 2s_1 x_1(t) \geq 0$ by the definition of $x_1(t)$. We will prove by contradiction that $\ddot{\lambda}_2(t) > 0$. Suppose not, then there exists $t \in [0, T]$ such that $\ddot{\lambda}_2(t) = \ddot{\mu}_2(t) = 0$ implies that $x_1(t) = y_1(t) = \frac{1}{2}b(T-t)^2$. This contradicts the condition of $x_1(t) + y_1(t) \leq \frac{1}{2}b(T-t)^2$.

Under control (36), the state variable $x_1(t)$ is

$$x_1(t) = \begin{cases} \frac{1}{2}\frac{s_1}{s_1+s_2}b\big((T-t)^2 - (t_d-t)^2\big), & \text{if } t < t_d, \\ \frac{1}{2}\frac{s_1}{s_1+s_2}b(T-t)^2, & \text{otherwise.} \end{cases} \tag{37}$$

The initial condition yields

$$x_1(0) + y_1(0) = \frac{1}{2}b(T^2 - t_d^2).$$

Plugging (37) into (34), we have

$$\begin{aligned} \dot{\lambda}_1(t) &= \begin{cases} -\frac{s_1 s_2}{s_1+s_2}b(T-t)^2 - \frac{s_1^2}{s_1+s_2}b(t_d-t)^2, & \text{if } t < t_d, \\ -\frac{s_1 s_2}{s_1+s_2}b(T-t)^2, & \text{otherwise;} \end{cases} \\ \lambda_1(0) &= \frac{1}{2\gamma}\big(x_1(0) + y_1(0)\big) = \frac{1}{4\gamma}b(T^2 - t_d^2). \end{aligned} \tag{38}$$

If $t \leq t_d$, the switching function $\lambda_2(t)$ is

$$\begin{aligned} \lambda_2(t) &= \lambda_2(0) - \int_{s=0}^{t} \lambda_1(s)ds \\ &= \lambda_2(0) - \int_{s=0}^{t}\left(\lambda_1(0) + \int_{\tau=0}^{s}\dot{\lambda}_1(\tau)d\tau\right)ds \\ &= -\frac{1}{4\gamma}b\left(T^2 - t_d^2\right)t + \frac{1}{12}\frac{s_1 s_2}{s_1+s_2}b\left(6T^2t^2 - 4Tt^3 + t^4\right) + \frac{1}{12}\frac{s_1^2}{s_1+s_2}b\left(6{t_d}^2t^2 - 4t_dt^3 + t^4\right). \end{aligned} \tag{39}$$

The condition that $\lambda_2(t_d) = 0$ yields either $t_d = 0$ or

$$Q(t_d) \equiv 2t_d^3 - 2Tt_d^2 + 3T^2 t_d - \frac{3}{\gamma s_1}\left(T^2 - t_d^2\right) = 0. \tag{40}$$

LEMMA 6 *There is a unique solution $t_d^* \in (0, T)$ to (40).*

Proof. Because $Q(t_d|t_d = 0) = -\frac{3}{\gamma s_1}T^2 < 0$ and $Q(t_d|t_d = T) = 3T^3 > 0$, there exists at least one solution to (40) in $(0, T)$. Additionally, we have $Q'(t_d) = 6(t_d - \frac{1}{3}T)^2 + \frac{7}{3}T^2 + \frac{6}{\gamma s_1}t_d > 0$ for any $t_d \in [0, T]$. Hence, this solution is unique.

By Lemmas 4, 5 and 6, we have that $\lambda_2(t) < 0$ for any $t \in (0, T)$, crosses zero at $t_d \in (0, T)$ and stays positive afterwards. Therefore, the proposed control (36) satisfies the necessary condition of the Maximum Principle.

Q.E.D.

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